Mathematics in the Modern World

Geometric Designs

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Geometric Designs

Geometry can help enhance ones artistic prowess as well as enrich ones own culture.

Learning outcomes

- Apply geometric concepts, especially isometries in describing and creating designs.
- Contribute to the enrichment of the Filipino culture and arts using concepts in geometry.

Fractals

(Aufmann, Lockwood, Nation, & Clegg, 2013, §7.7)

A fractal *is a geometric figure in which a self-similar motif repeats itself on an ever-diminishing scale.*

Fractals are generally constructed by using iterative processes in which the fractal is more closely approximated as a repeated cycle of procedures is performed.

No. 5, 1948 by Jackson Pollock

(Vogel, 2006)

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(Vogel, 2006)

In 2006, this painting was reportedly sold for about $140 million, the highest sum ever known at that time to have been paid for a painting.

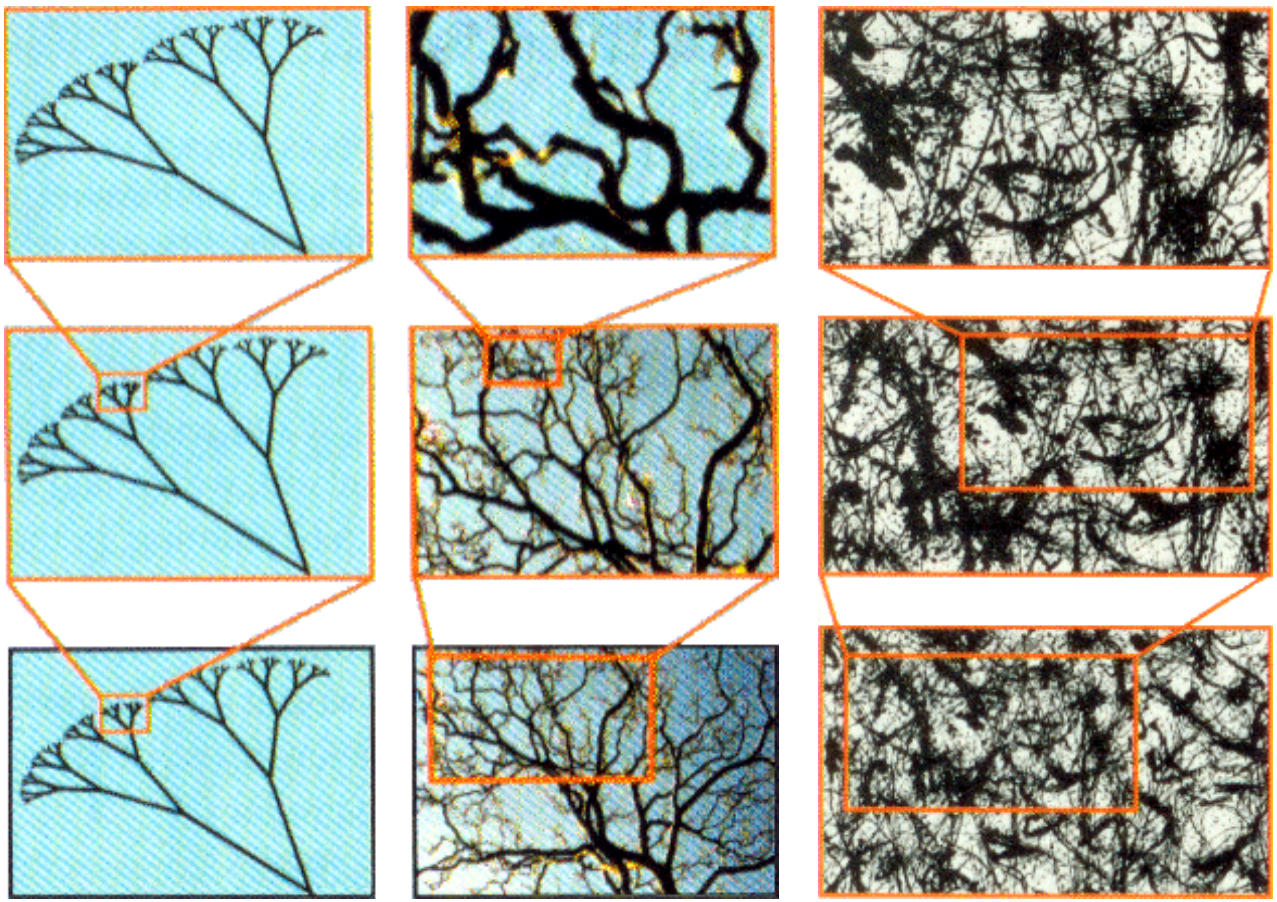
Fractal analysis of Pollock's drip paintings

(Taylor, Micolich, & Jonas, 1999; Taylor, 2002)

Scientific objectivity proves to be an essential tool for determining the fundamental content of the abstract paintings produced by Jackson Pollock in the late 1940s. Pollock dripped paint from a can onto vast canvases rolled out across the floor of his barn. Although this unorthodox technique has been recognized as a crucial advancement in the evolution of modern art, the precise quality and significance of the patterns created are controversial. Here we describe an analysis of Pollock's patterns which shows, first, that they are fractal, reflecting the fingerprint of nature, and, second, that the fractal dimensions increased during Pollock's career.

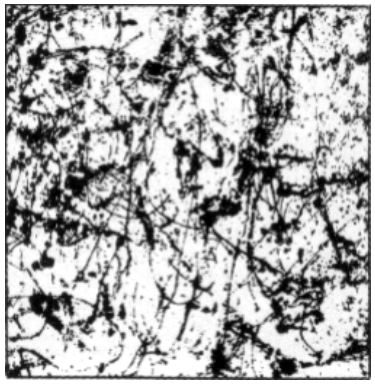
Fractals are self-similar

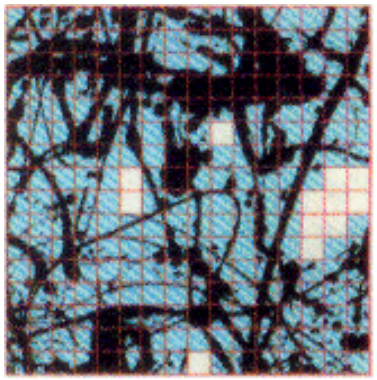
(Taylor, 2002)

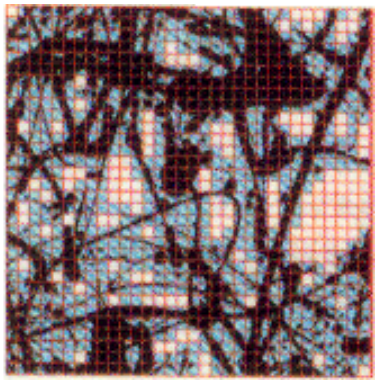


The fractal character of Pollock's paintings

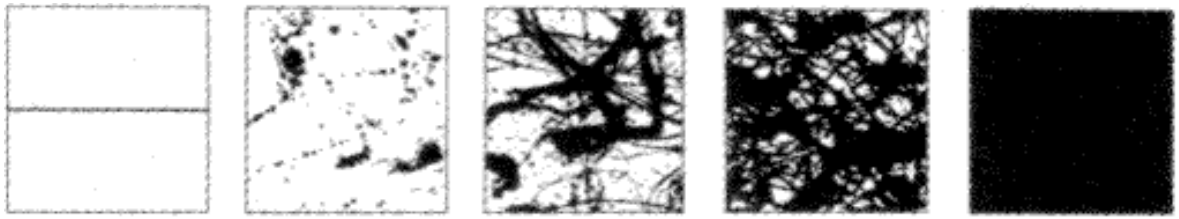
(Taylor, 2002)

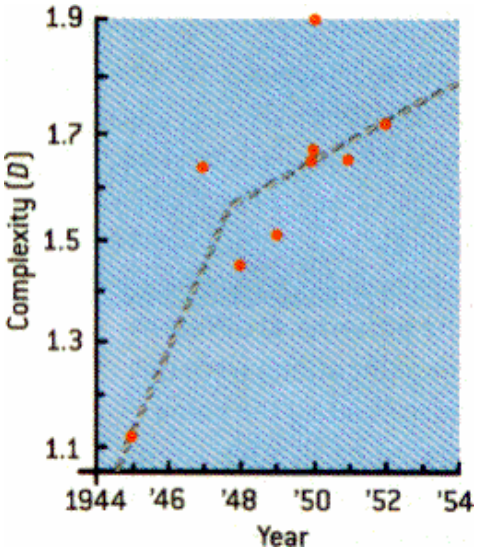






The painting is scanned into a computer. It is separated into its different colored patterns, then covered with a computer-generated mesh of identical squares. The computer analyzes which squares are occupied and which are empty. This is done for different mesh sizes. The patterns were found to be fractal over the entire size range.





Studying the paintings chronologically showed that the complexity of the fractal patterns, D, increased as Pollock refined his technique.

One D value is clearly an outlier—1.9 in 1950, a work that Pollock later destroyed. He may have thought this image was too dense or too complex and subsequently scaled back.

Mathematical and anthropological analysis of northern Luzon funeral textile

(De Las Peñas & Salvador-Amores, 2016, p. 91)

An *isometry* is a geometric transformation that keeps the distance between points unchanged. (A *symmetry* is an isometry which sends a pattern to itself.)

(a) (b) (c) (d)

Mathematical and anthropological analysis of northern Luzon funeral textile (continuation) (De Las Peñas & Salvador-Amores, 2016, p. 91)

There are exactly four types of isometries in the plane:

(a) A *translation* moves every point of the plane through a fixed distance in a particular direction specified by a given vector

(b) A *reflection* moves every point of the plane to its mirror image about a fixed line called an axis of reflection

(c) A *rotation* moves every point of the plane through a fixed angle about a fixed point called the center of rotation

(d) A *glide reflection* is a combination of a translation and a reflection, defined by specifying a reflection axis and a translation vector parallel to the axis of reflection

Mathematical and anthropological analysis of northern Luzon funeral textile (continuation) (De Las Peñas & Salvador-Amores, 2016, pp. 91–92)

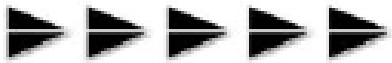
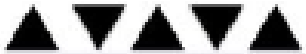
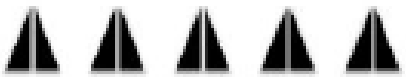
There are three (non-trivial) classes of symmetrical patterns that may be found in textile:

1. A *finite pattern* is a repeated pattern that does not admit any translational symmetries. It admits either only rotational symmetries or both rotational and reflectional symmetries
2. A *frieze pattern* is a symmetric pattern along a strip that has translational symmetries in one direction
3. A *plane crystallographic pattern* is a repeated pattern that has translational symmetries in two directions

Mathematical and anthropological analysis of northern Luzon funeral textile (continuation) (De Las Peñas & Salvador-Amores, 2016, pp. 92)

The set consisting of all symmetries of a finite pattern, frieze pattern or a plane crystallographic pattern form an algebraic structure called a group *under the operation composition of isometries. This is called the* symmetry group *of the respective pattern. There are seven types of frieze patterns and seventeen types of plane crystallographic patters known; each is classified based on its symmetry group or the symmetries the pattern admits.*

Mathematical and anthropological analysis of northern Luzon funeral textile (continuation) (De Las Peñas & Salvador-Amores, 2016, Table 1)

Frieze Pattern	Symmetry group
	mm
	1m
	mg
	1g
	12
	m1
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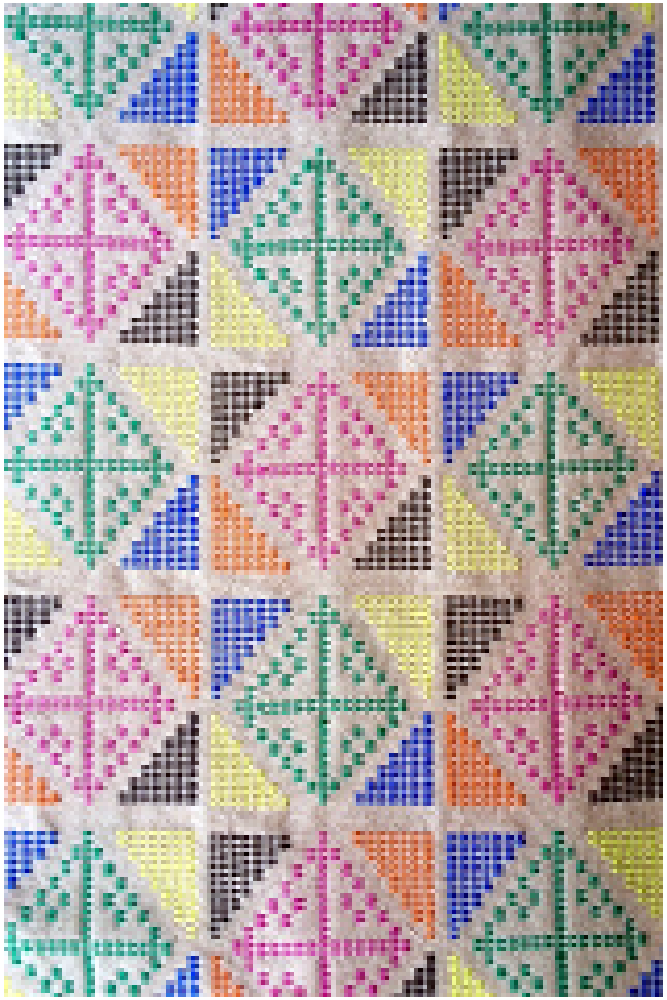
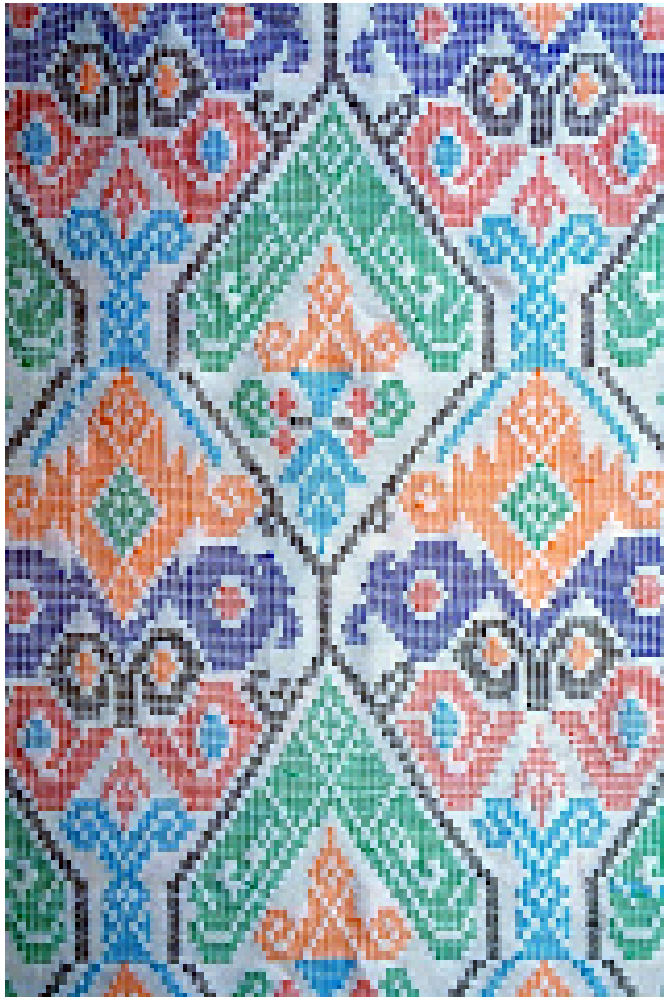
Yakan textile weaving designs

(Haute Culture, 2016)

Yakan textile weaving designs (continuation)

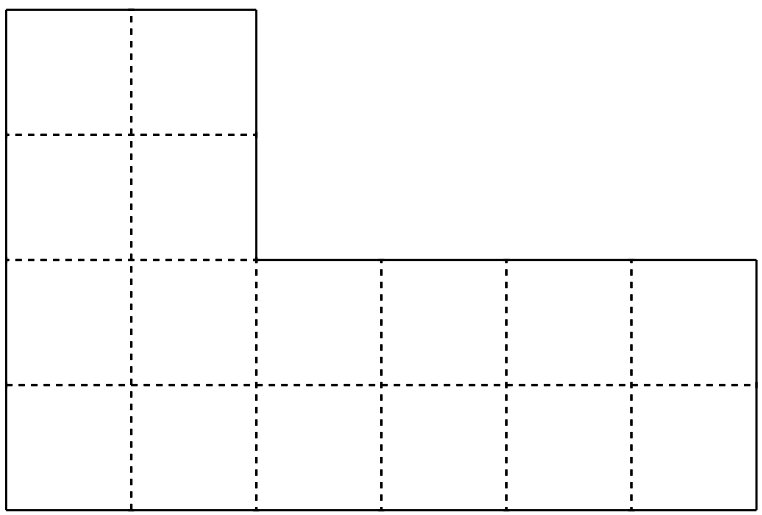
(Haute Culture, 2016)

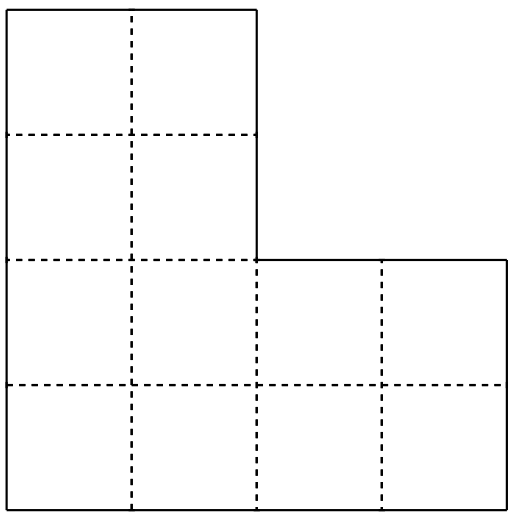




Sample exam question

Dissect the rep-tiles below into four smaller (same-sized) copies of themselves.





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