

Mathematics in the Modern World

Problem Solving and Reasoning

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Problem Solving and Reasoning

Mathematics is not just about numbers; much of it is problem solving and reasoning

Learning outcomes

- ▶ Use different types of reasoning to justify statements and arguments made about mathematics and mathematical concepts.
- ▶ Write clear and logical proofs.
- ▶ Solve problems involving patterns and recreational problems following Polya's four steps.
- ▶ Organize one's methods and approaches for proving and solving problems.

Polya's four phases in solving problems

(Polya, 1957, p. 5)

Trying to find the solution, we may repeatedly change our point of view, our way of looking at the problem. We have to shift our position again and again. Our conception of the problem is likely to be rather incomplete when we start the work; our outlook is different when we have made more progress; it is again different when we have almost obtained the solution.

Polya's four phases in solving problems (continuation)

(Polya, 1957, pp. 5–6)

In order to group conveniently the questions and suggestions of our list, we shall distinguish four phases of the work. First, we have to understand the problem; we have to see clearly what is required. Second, we have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution, to make a plan. Third, we carry out our plan. Fourth, we look back at the completed solution, we review and discuss it.

Can you solve “Einstein’s Riddle”?

A TED-Ed Original lesson by Dan Van der Vieren, 2015

(5:12): https://youtu.be/1rDVz_Fb6HQ

See also <http://ed.ted.com/lessons/can-you-solve-einstein-s-riddle-dan-van-der-ieren>.

Can you solve the control room riddle?

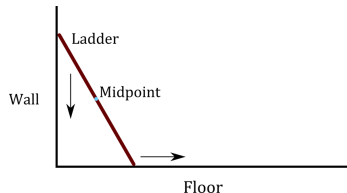
A TED-Ed Original lesson by Dennis Shasha, 2016

(4:01): <https://youtu.be/3mbdiky5dLw>

See also <http://ed.ted.com/lessons/can-you-solve-the-control-room-riddle-dennis-shasha>.

A sample interview question for mathematics

(University of Oxford, n.d.)



“Imagine a ladder leaning against a vertical wall with its feet on the ground. The middle rung of the ladder has been painted a different colour on the side, so that we can see it when we look at the ladder from the side on. What shape does that middle rung trace out as the ladder falls to the floor?”

A solution can be found in Gutenmacher and Vasilyev (2004, p. 1).

Sample exam question

The following problem is paraphrased from page 3 of V. I. Arnold's book *Problems for children from 5 to 15*:

Explain what is wrong with the following mathematics word problem:

The hypotenuse of a right-angled triangle is 10 inches, the altitude dropped onto it is 6 inches. Find the area of the triangle.

Hint: Any right triangle can be inscribed in a semicircle with the hypotenuse of the triangle as the diameter of the circle.

- Gutenmacher, V., & Vasilyev, N. B. (2004). *Lines and curves: A practical geometry handbook*. Boston, MA: Birkhäuser.
- Polya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.
- University of Oxford. (n.d.). *Sample interview questions*. Retrieved April 28, 2017, from <http://www.ox.ac.uk/admissions/undergraduate/applying-to-oxford/interviews/sample-interview-questions>