Mathematics in the Modern World

Mathematics in Our World

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Mathematics in Our World

Mathematics is a useful way to think about nature and our world

Learning outcomes

- Identify patterns in nature and regularities in the world.
- Articulate the importance of mathematics in one's life.
- Argue about the nature of mathematics, what it is, how it is expressed, represented, and used.
- Express appreciation for mathematics as a human endeavor.

Nature by Numbers

A short movie by Cristóbal Vila, 2010

Original (3:44): `https://vimeo.com/9953368`

Alternative soundtrack (4:04): `https://vimeo.com/29379521`

Fibonacci sequence

(Vila, 2016)

0+1=**1**

0 1+1=**2**

0 1 1+2=**3**

0 1 1 2+3=**5**

0 1 1 2 3+5=**8**

0 1 1 2 3 5+8=**13**

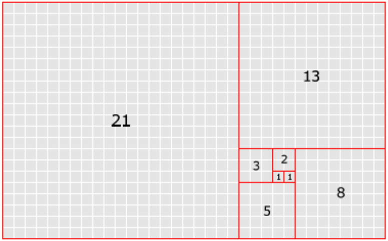
etc

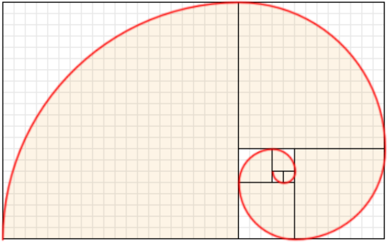
0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181

The Fibonacci sequence "is an infinite sequence of natural numbers where the first value is 0, the next is 1 and, from there, each amount is obtained by adding the previous two."

Fibonacci spiral

(Vila, 2016)

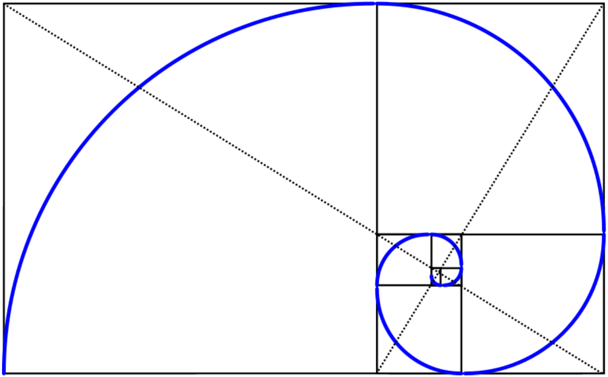




Circular arcs connect the opposite corners of squares in the Fibonacci tiling.

Golden spiral

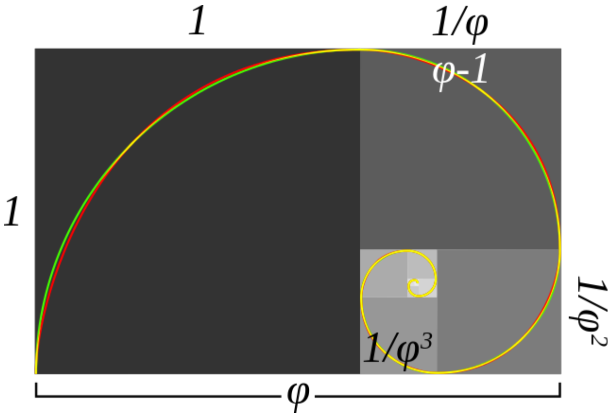
(Golden spiral in rectangles, 2008)



$r = \varphi^{2\theta/\pi}$ where θ is in radians and $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio

Comparison of spirals

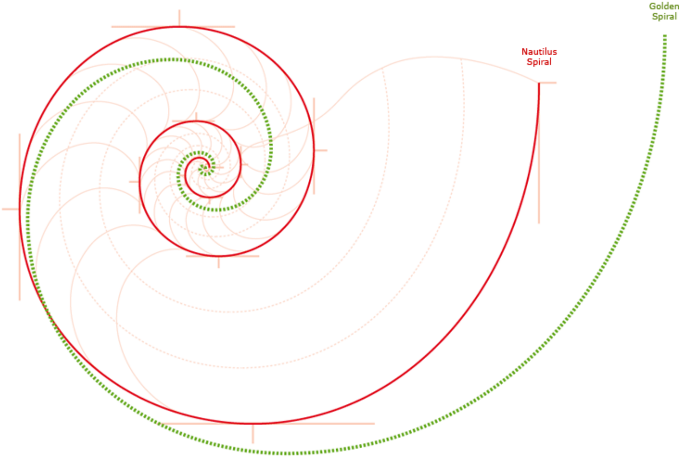
(Approximate and true Golden Spirals, 2009)



Quarter-circles in green, golden spiral in red, overlaps in yellow

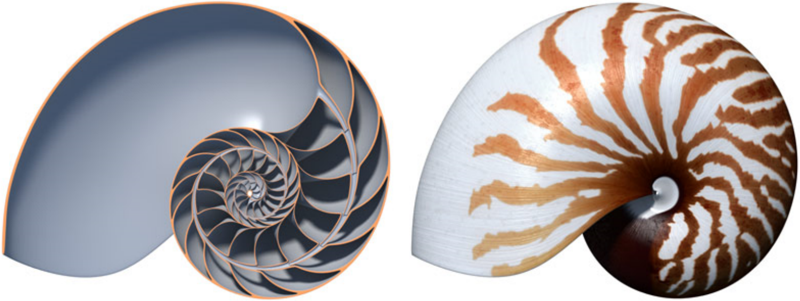
Nautilus spiral

(Vila, 2016)



Vila's mistake

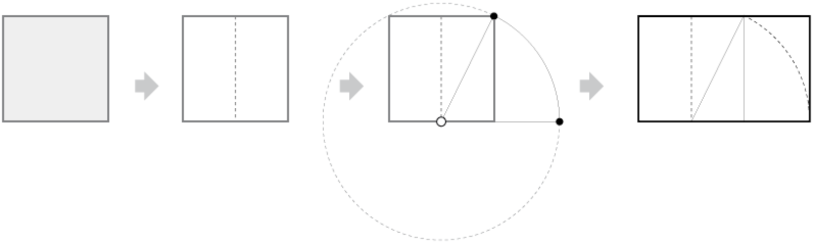
(Vila, 2016)



Vila admits he made a mistake in the animation for the Nautilus shell. (It is neither a Fibonacci spiral nor a golden spiral.)

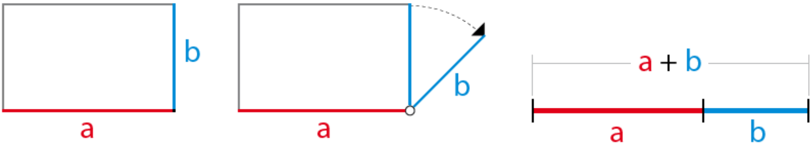
Golden rectangle

(Vila, 2016)



Golden ratio

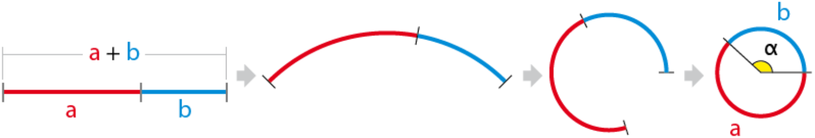
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = \mathbf{1.61803399...}$$

Golden angle

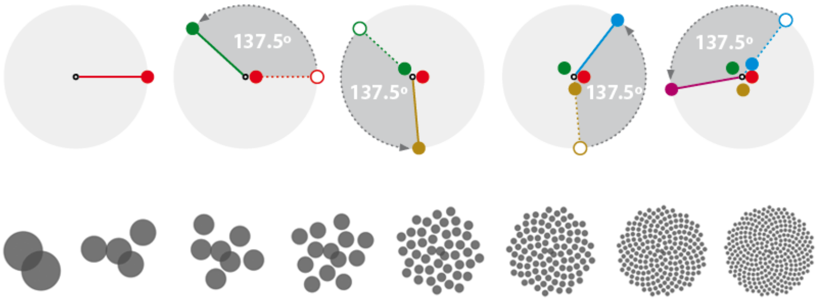
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots \rightarrow \alpha = \mathbf{137.507764^\circ\ldots} \ \boxed{\mathbf{\sim 137.5^\circ}}$$

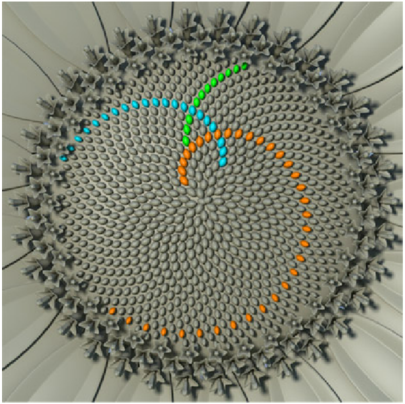
Golden angle and arrangement of sunflower seeds

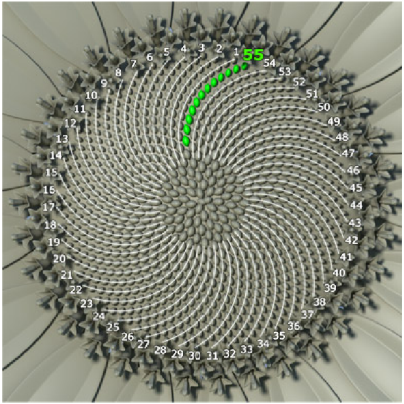
(Vila, 2016)



Fibonacci numbers and arrangement of sunflower seeds

(Vila, 2016)

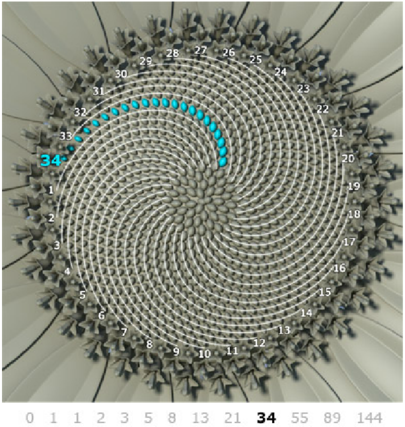


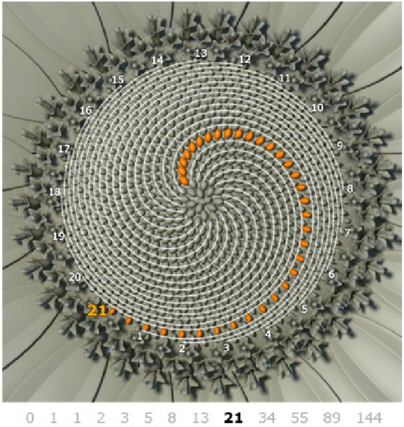


0 1 1 2 3 5 8 13 21 34 **55** 89 144

Fibonacci numbers and arrangement of sunflower seeds

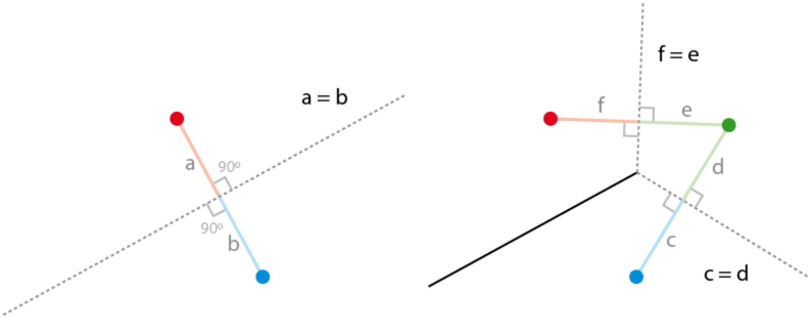
(continuation) (Vila, 2016)





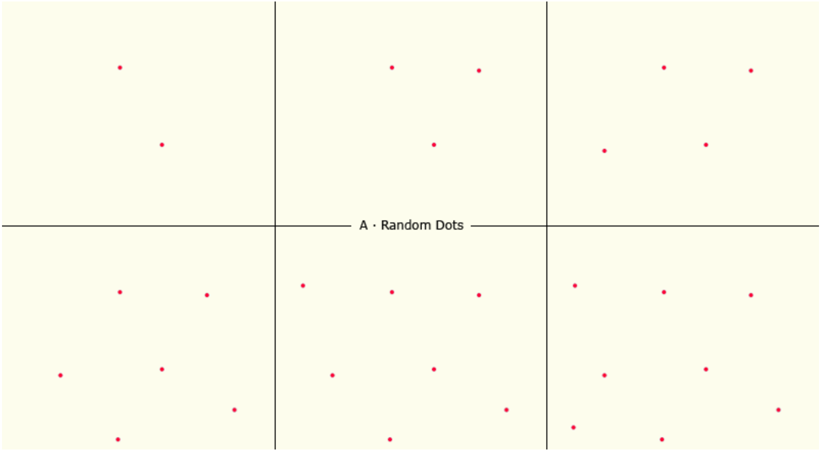
Delaunay triangulation and Voronoi diagram

(Vila, 2016)



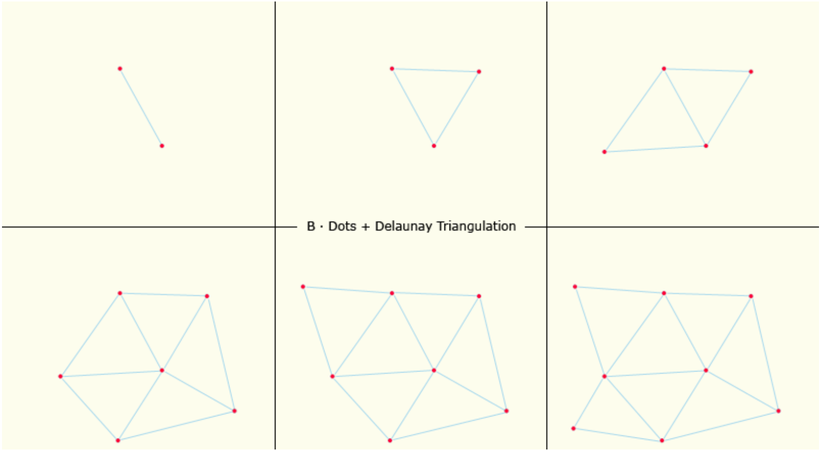
Delaunay triangulation and Voronoi diagram

(continuation) (Vila, 2016)



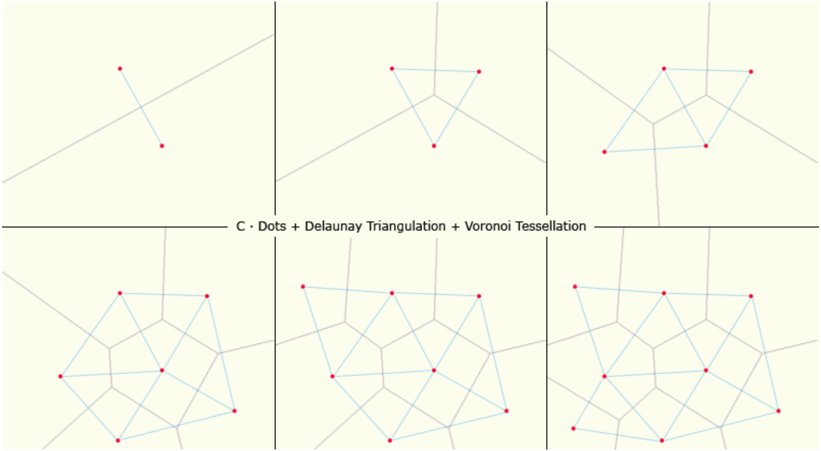
Delaunay triangulation and Voronoi diagram

(continuation) (Vila, 2016)



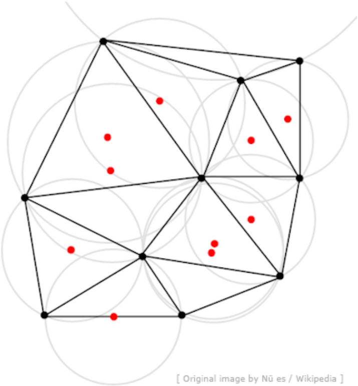
Delaunay triangulation and Voronoi diagram

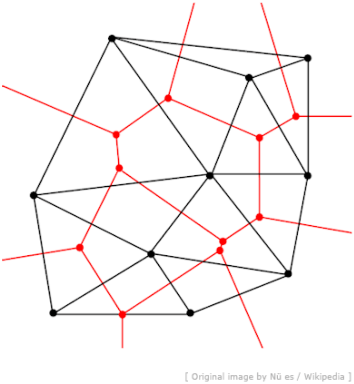
(continuation) (Vila, 2016)



Delaunay condition

(Vila, 2016; Delaunay triangulation, 2017)

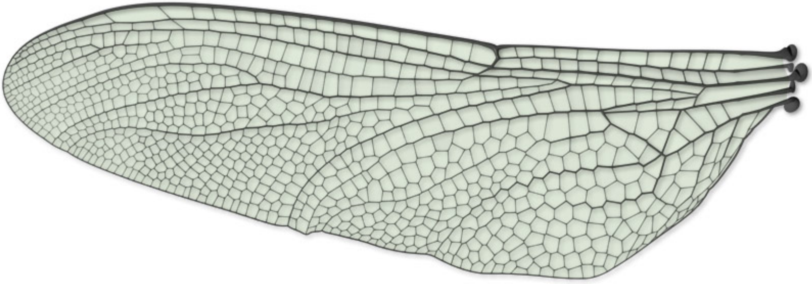




A Delaunay triangulation for a set of points in a plane is a triangulation such that no point is inside the circumcircle of any triangle.

Dragonfly wings

(Vila, 2016)



Relationship between golden ratio and Fibonacci sequence

(Vila, 2016; Golden ratio, 2017)

1/1	=	1
2/1	=	2
3/2	=	1.5
5/3	=	1.66666666666
8/5	=	1.6
13/8	=	1.625
21/13	=	1.61538461538
34/21	=	1.61904761905
55/34	=	1.61764705882
89/55	=	1.61818181818
—		
Phi	**=**	**1.6180339887...**

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \quad \text{for } n > 1, n \in \mathbb{Z}$$

$$F_n = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$$

Sample exam questions

1. Let $\varphi = \frac{1}{2}\left(1+\sqrt{5}\right)$. Show that $\varphi = \varphi^2 - 1 = \dfrac{1}{\varphi} + 1$.
2. Let $\phi = \frac{1}{2}\left(1-\sqrt{5}\right)$. Show that $\phi = \phi^2 - 1 = \dfrac{1}{\phi} + 1$.
3. Explain why you think the golden ratio is defined to be $\varphi = \frac{1}{2}\left(1+\sqrt{5}\right)$ and *not* $\phi = \frac{1}{2}\left(1-\sqrt{5}\right)$.

Mathematics is a useful way to think about nature

(Stewart, 1995, p. 19)

Whatever the reasons, mathematics definitely is a useful way to think about nature. What do we want it to tell us about the patterns we observe? There are many answers. We want to understand how they happen; to understand why *they happen, which is different; to organize the underlying patterns and regularities in the most satisfying way; to predict how nature will behave; to control nature for our own ends; and to make practical use of what we have learned about our world. Mathematics helps us to do all these things, and often it is indispensable.*

Sample reading assignment

Read Stewart (1995) and be ready to answer the following discussion questions.

- Which sentence or paragraph in the book is your favorite? Why?
- Is there any statement or point of view in the book that you disagree with?
- How would you summarize each of the nine chapters (in one, two, or three sentences per chapter)?
- How does Stewart differentiate the external aspects of mathematics from the internal aspects of mathematics?
- What term does Stewart use to describe his dream of an effective mathematical theory of form and the emergence of pattern?

Sample assignment

Using Stewart (1995) as your reference, write an essay of around 250 to 350 words answering exactly one of the following questions.

- How did Stewart explain why numbers from the Fibonacci series appear when some features in plants are counted?
- Leonhard Euler got into an argument with Daniel Bernoulli because their solutions to the one-dimensional wave equation differed. How did Stewart explain the argument's resolution?
- How did Stewart use Poincaré's concept of a phase space to explain why tides are predictable but weather is not?
- How did Stewart use coupled oscillator networks to model animal gaits?

Is math discovered or invented?

A TED-Ed Original lesson by Jeff Dekofsky, 2014

(5:11): https://youtu.be/X_xR5Kes4Rs

See also http://ed.ted.com/lessons/is-math-discovered-or-invented-jeff-dekofsky.

Philosophy of mathematics

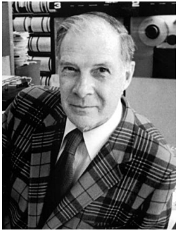
(Philosophy of Mathematics, 2017)

"*Mathematical realism* [...] holds that mathematical entities exist independently of the human mind. Thus humans do not invent mathematics, but rather discover it [...]." One form of mathematical realism is Platonism.

"*Mathematical anti-realism* generally holds that mathematical statements have truth-values, but that they do not do so by corresponding to a special realm of immaterial or non-empirical entities." One form of mathematical anti-realism is formalism.

Platonism

"*Mathematical Platonism* is the form of realism that suggests that mathematical entities are abstract, have no spatiotemporal or causal properties, and are eternal and unchanging." (Philosophy of Mathematics, 2017)



(Richard Hamming, 2013)

"Very few of us in our saner moments believe that the particular postulates that some logicians have dreamed up create the numbers—no, most of us believe that the real numbers are simply there and that it has been an interesting, amusing, and important game to try to find a nice set of postulates to account for them." (Hamming, 1980, p. 85)

Formalism

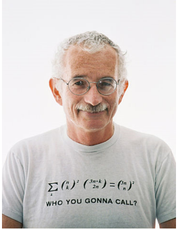
"*Formalism* holds that mathematical statements may be thought of as statements about the consequences of certain string manipulation rules." (Philosophy of Mathematics, 2017)

(David Hilbert, 2017)

"Mathematics, according to David Hilbert (1862-), is a game played according to certain simple rules with meaningless marks on paper." (Stabler, 1935, p. 24)

Ultrafinitism

"*[C]onstructivism* involves the regulative principle that only mathematical entities which can be explicitly constructed in a certain sense should be admitted to mathematical discourse. [...]
Finitism is an extreme form of constructivism, according to which a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps. [...]
Ultrafinitism is an even more extreme version of finitism, which rejects not only infinities but finite quantities that cannot feasibly be constructed with available resources." (Philosophy of Mathematics, 2017)



(Doron Zeilberger, 2007)

"What is completely meaningless is any kind of *infinite*, actual or potential. So I deny even the existence of the Peano axiom that every integer has a successor. [...]
The phrase 'for *all* positive integers' is meaningless. [...]
Similarly, Euclid's statement: 'There are infinitely many primes' is meaningless." (Zeilberger, 2001, p. 5)

Constructive and nonconstructive proofs

Theorem

There exist irrational numbers a and b such that a^b is rational.

Nonconstructive proof.

Consider $\sqrt{2}^{\sqrt{2}}$. If it is rational, then the proof is complete. If it is not rational, then take $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$ so that $a^b = 2$. $\square$

Constructive proof (sketch).

Take $a = \sqrt{2}$ and $b = 2\log_2 3$. $\square$

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