Interdisciplinary Approach to Teaching 'Mathematics in the Modern World'

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Mathematics in the Modern World

Training of GE trainors, Ateneo de Manila University, October 10–28, 2017

Matematika sa Makabagong Daigdig

Training of GE trainors, Ateneo de Manila University, October 10–28, 2017

Mathematics in the Modern World

Second-generation GE faculty training, Ateneo de Naga University, April 24, 2017–May 13, 2017

New General Education Curriculum

The new general education curriculum (CHED, 2013) will soon replace the one described in CHED (1996).

It is intended to generally start academic year 2018–2019, but "[i]n light of the transition models implemented by some private basic education schools where they re-label the grades, such that students will graduate as early as 2016, the new GE shall be implemented earlier than 2018" (CHED, 2013, p. 10).

CMO no. 20, s. 2013 (CHED, 2013) is "a set of minimum standards for the general education component of all degree programs that applies to private and public Higher Education Institutions in the country."

Mathematics in the Modern World/ Matematika sa Makabagong Daigdig

Description (CHED, 2013)

Nature of mathematics, appreciation of its practical, intellectual, and aesthetic dimensions, and application of mathematical tools in daily life./
Mga elemento ng matematika, pagpapahalaga sa mga praktikal, intelektuwal, at estetikong dimensiyon nito; at gamit ng matematika sa araw araw na buhay.

Mathematics in the Modern World/ Matematika sa Makabagong Daigdig

Brief explanation (CHED, 2013)

The course begins with an introduction to the nature of mathematics as an exploration of patterns (in nature and the environment) and as an application of inductive and deductive reasoning. By exploring these topics, students are encouraged to go beyond the typical understanding of mathematics as merely a bunch of formulas, but as a source of aesthetics in patterns of nature, for example, and a rich language in itself (and of science) governed by logic and reasoning.

Mathematics in the Modern World/ Matematika sa Makabagong Daigdig

Brief explanation (continuation) (CHED, 2013)

The course then proceeds to survey ways in which mathematics provides a tool for understanding and dealing with various aspects of present day living, such as managing personal finances, making social choices, appreciating geometric designs, understanding codes used in data transmission and security, and dividing limited resources fairly. These aspects will provide opportunities for actually doing mathematics in a broad range of exercises that bring out the various dimensions of mathematics as a way of knowing and test the students' understanding and capacity.

Sample syllabi

According to CHED (2016), "the sample or suggested course syllabi can be used as guides," and "[p]ublic and private HEIs may adopt the sample or suggested course syllabi in the teaching and in the delivery of the content of the new GEC. For record purposes, the HEIs shall inform the CHED Regional Offices (CHEDROs) of their implementation of the new GEC."

Mathematics in the Modern World:
`http://api.ched.ph/api/v1/download/2760`

Matematika sa Makabagong Daigdig:
`http://api.ched.ph/api/v1/download/2775`

Mathematics in the Modern World

The Nature of Mathematics

- Mathematics in Our World
- Mathematical Language and Symbols
- Problem Solving and Reasoning

Mathematics as a Tool

- Data Management
- Geometric Designs
- Codes
- Linear Programming
- The Mathematics of Finance
- Apportionment and Voting
- Logic
- The Mathematics of Graphs
- Mathematical Systems

Mathematics in Our World

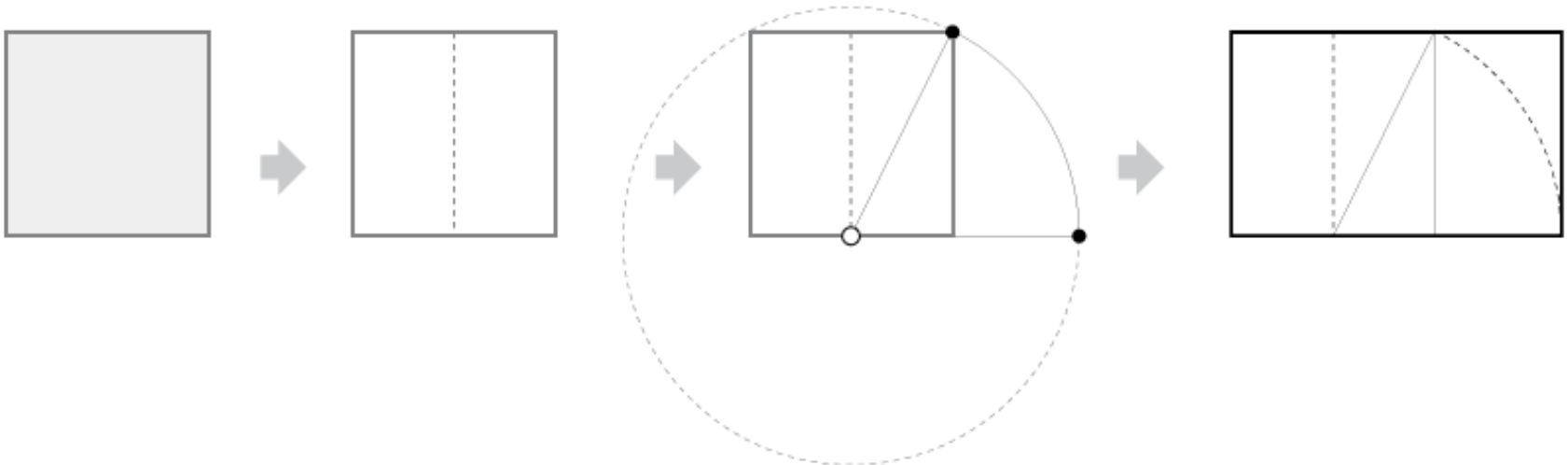
Mathematics is a useful way to think about nature and our world

Learning outcomes

- Identify patterns in nature and regularities in the world.
- Articulate the importance of mathematics in one's life.
- Argue about the nature of mathematics, what it is, how it is expressed, represented, and used.
- Express appreciation for mathematics as a human endeavor.

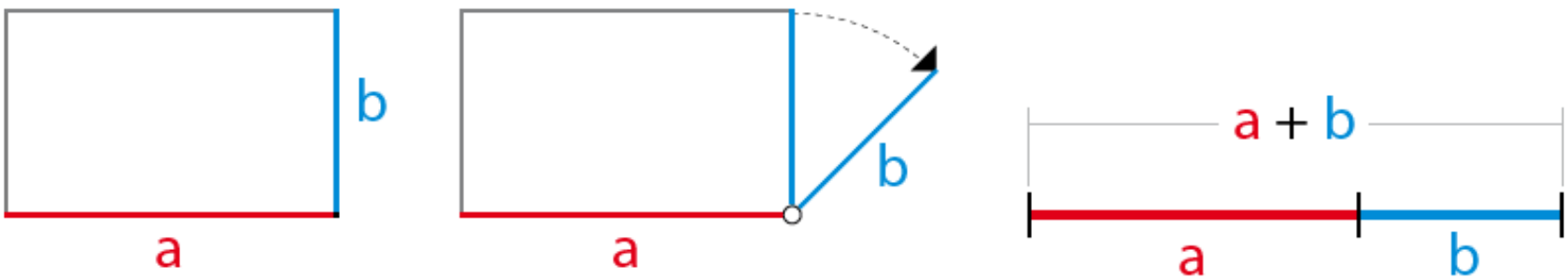
Golden rectangle

(Vila, 2016)



Golden ratio

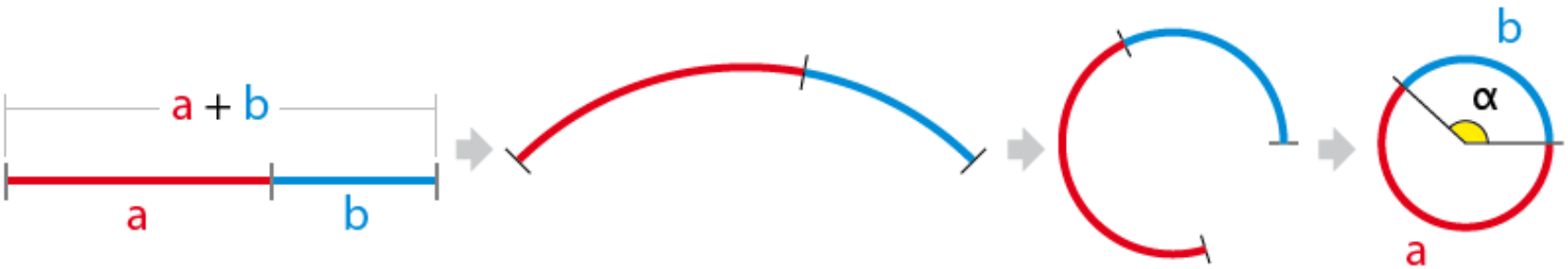
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots$$

Golden angle

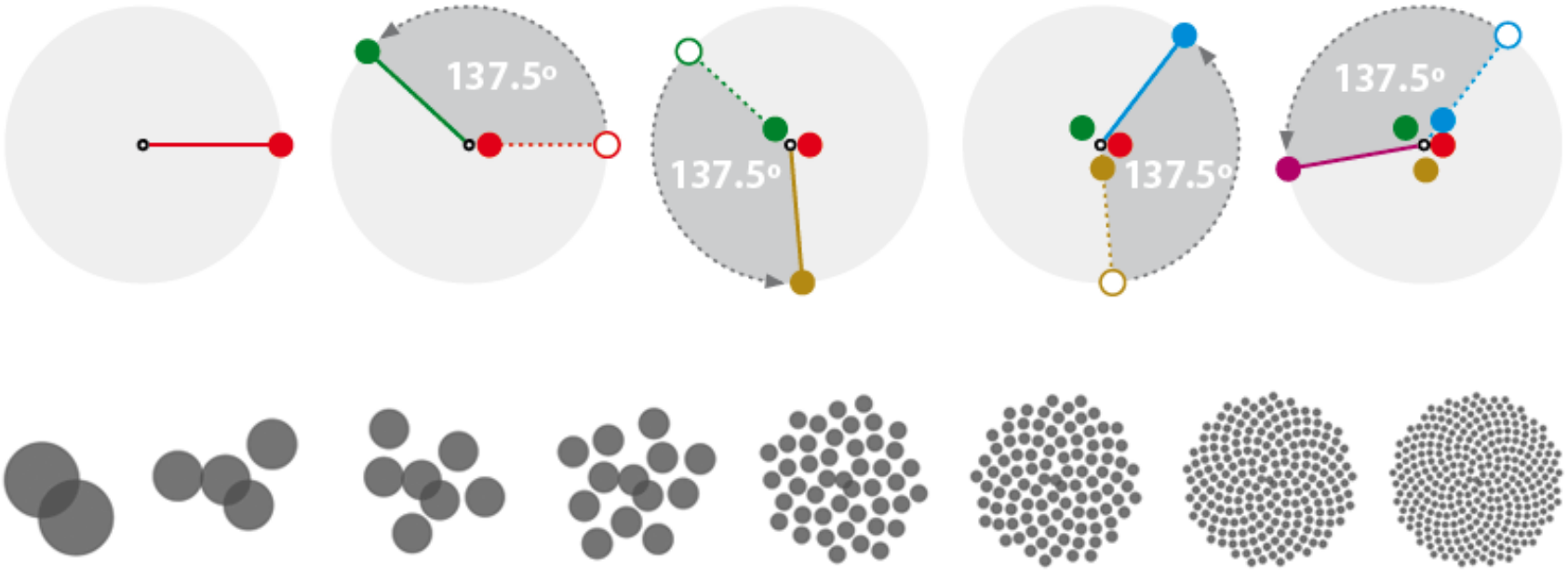
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots \Rightarrow \alpha = 137.507764^{\circ}\ldots \sim 137.5^{\circ}$$

Golden angle and arrangement of sunflower seeds

(Vila, 2016)



Fibonacci sequence

(Vila, 2016)

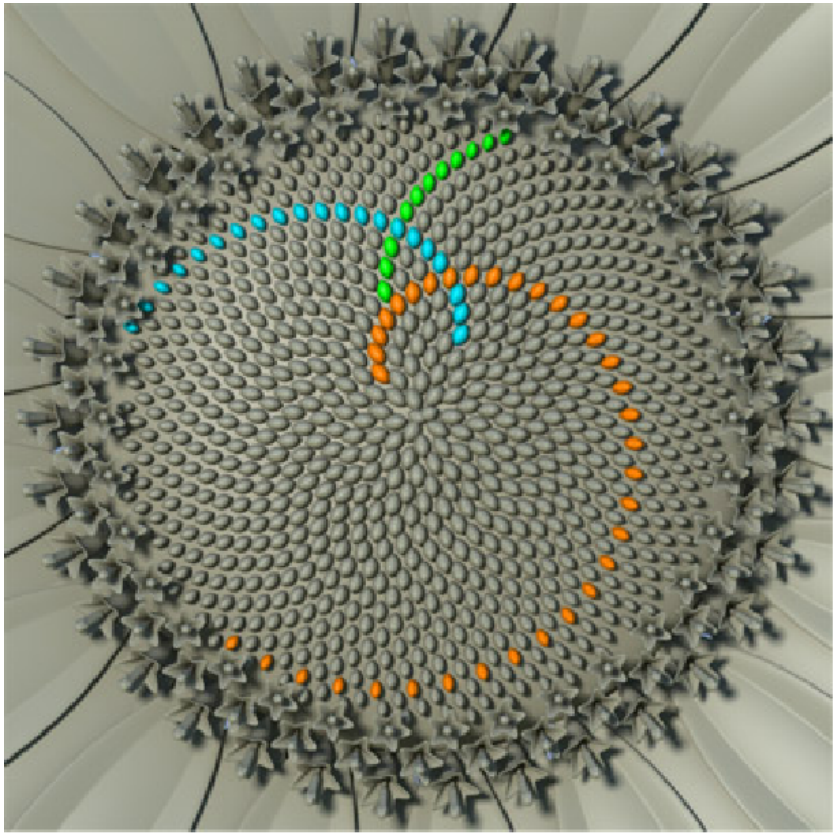
0+1=**1**
0 1+1=**2**
0 1 1+2=**3**
0 1 1 2+3=**5**
0 1 1 2 3+5=**8**
0 1 1 2 3 5+8=**13**
etc

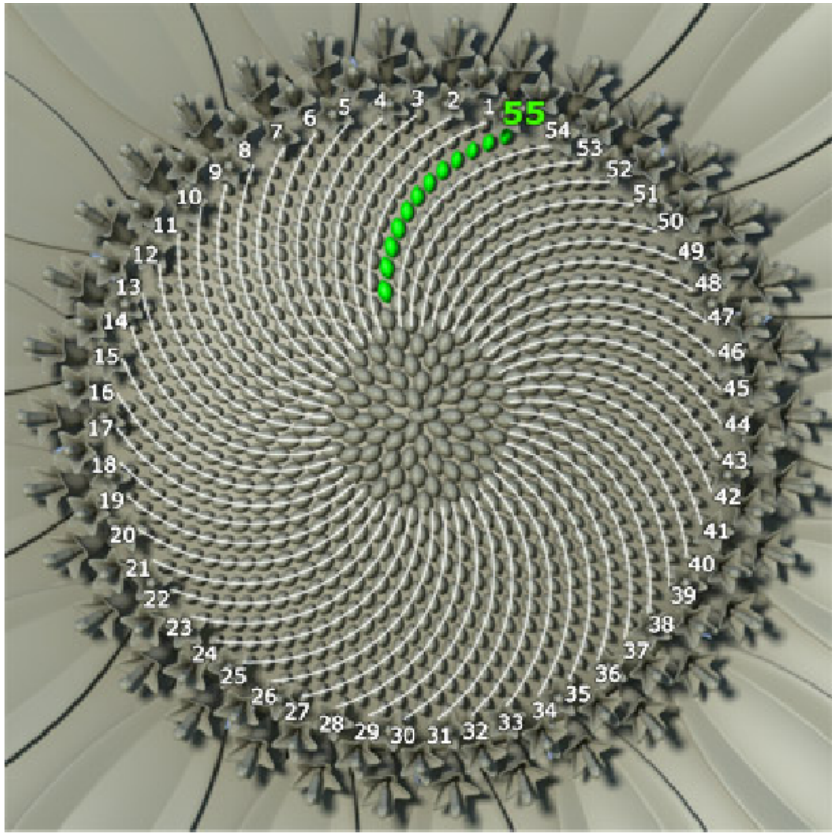
0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181

The Fibonacci sequence "is an infinite sequence of natural numbers where the first value is 0, the next is 1 and, from there, each amount is obtained by adding the previous two."

Fibonacci numbers and arrangement of sunflower seeds

(Vila, 2016)

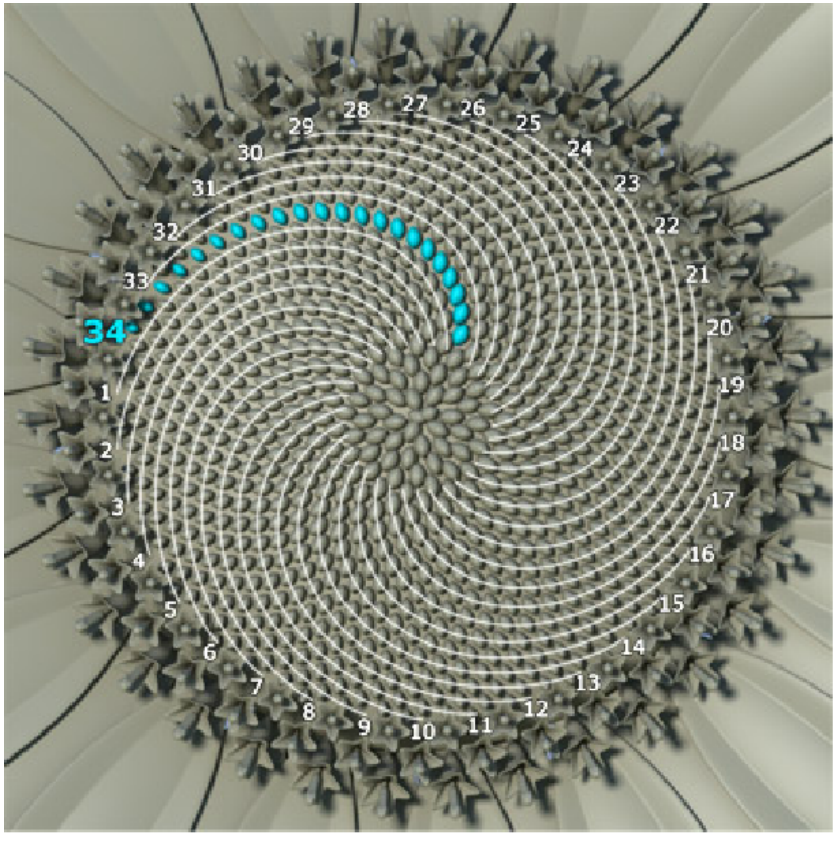


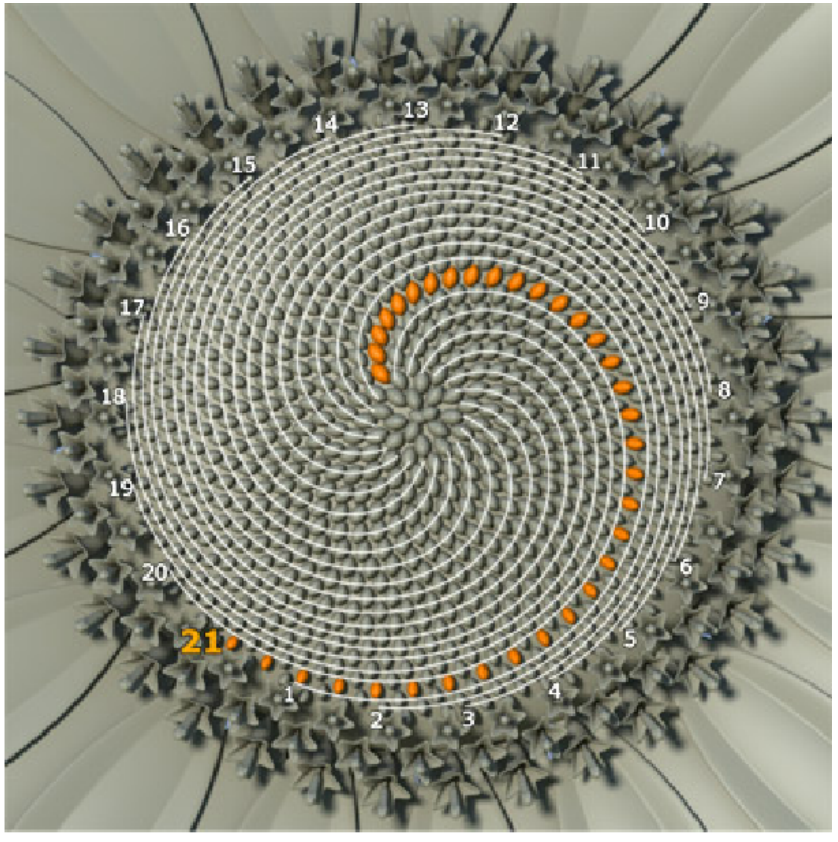


0 1 1 2 3 5 8 13 21 34 **55** 89 144

Fibonacci numbers and arrangement of sunflower seeds

(continuation) (Vila, 2016)





Philosophy of mathematics

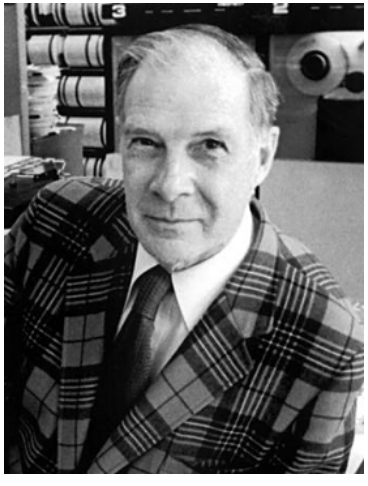
(Philosophy of Mathematics, 2017)

"*Mathematical realism* [...] holds that mathematical entities exist independently of the human mind. Thus humans do not invent mathematics, but rather discover it [...]." One form of mathematical realism is Platonism.

"*Mathematical anti-realism* generally holds that mathematical statements have truth-values, but that they do not do so by corresponding to a special realm of immaterial or non-empirical entities." One form of mathematical anti-realism is formalism.

Platonism

"*Mathematical Platonism* is the form of realism that suggests that mathematical entities are abstract, have no spatiotemporal or causal properties, and are eternal and unchanging." (Philosophy of Mathematics, 2017)



(Richard Hamming, 2013)

"Very few of us in our saner moments believe that the particular postulates that some logicians have dreamed up create the numbers—no, most of us believe that the real numbers are simply there and that it has been an interesting, amusing, and important game to try to find a nice set of postulates to account for them." (Hamming, 1980, p. 85)

Formalism

"*Formalism* holds that mathematical statements may be thought of as statements about the consequences of certain string manipulation rules." (Philosophy of Mathematics, 2017)

(David Hilbert, 2017)

"Mathematics, according to David Hilbert (1862-), is a game played according to certain simple rules with meaningless marks on paper." (Stabler, 1935, p. 24)

Mathematical Language and Symbols

Like any language, mathematics has its own symbols, syntax, and rules

Learning outcomes

- Discuss the language, symbols, and conventions of mathematics.
- Explain the nature of mathematics as a language.
- Perform operations on mathematical expressions correctly.
- Acknowledge that mathematics is a useful language.

Implications

(Jamison, 2000)

If a figure is a square, then it is a rectangle.	Hypothetical
A figure is a square only if it is a rectangle.	
A figure is a rectangle whenever it is a square.	
All squares are rectangles.	Categorical
For a figure to be a square, it must necessarily be a rectangle.	Necessity
A sufficient condition for a figure to be a rectangle is that it be a square.	Sufficiency
A figure cannot be a square and fail to be a rectangle.	Conjunctive
A figure is either a rectangle or it is not a square.	Disjunctive

Quantifiers

(Gowers, 2008, p. 14)

Quantifiers in ordinary speech can be ambiguous.

1. Nothing is better than lifelong happiness.
2. But a cheese sandwich is better than nothing.
3. Therefore, a cheese sandwich is better than lifelong happiness.

The word "nothing" is used differently in (1) and in (2).

1. ~~(To have) nothing is better than (to have) lifelong happiness.~~
2. But (to have) a cheese sandwich is better than (to have) nothing.

Quantifiers (continuation)

(Gowers, 2008, p. 14)

Quantifiers in ordinary speech can be ambiguous.

1. Everybody likes at least one drink, namely water.
2. Everybody likes at least one drink; I myself go for red wine.

The clause "Everybody likes at least one drink" is used differently in (1) and in (2).

1. There exists a drink D such that, for every person P, P likes D.
2. For every person P there exists a drink D such that P likes D.

Geometric Designs

Geometry can help enhance ones artistic prowess as well as enrich ones own culture.

Learning outcomes

- Apply geometric concepts, especially isometries in describing and creating designs.
- Contribute to the enrichment of the Filipino culture and arts using concepts in geometry.

Mathematical and anthropological analysis of northern Luzon funeral textile (De Las Peñas & Salvador-Amores, 2016, p. 91)

An *isometry* is a geometric transformation that keeps the distance between points unchanged. (A *symmetry* is an isometry which sends a pattern to itself.)

(a) (b) (c) (d)

Mathematical and anthropological analysis of northern Luzon funeral textile (continuation) (De Las Peñas & Salvador-Amores, 2016, p. 91)

There are exactly four types of isometries in the plane:

(a) A *translation* moves every point of the plane through a fixed distance in a particular direction specified by a given vector

(b) A *reflection* moves every point of the plane to its mirror image about a fixed line called an axis of reflection

(c) A *rotation* moves every point of the plane through a fixed angle about a fixed point called the center of rotation

(d) A *glide reflection* is a combination of a translation and a reflection, defined by specifying a reflection axis and a translation vector parallel to the axis of reflection

Yakan textile weaving designs

(Haute Culture, 2016)

Codes

The utility of mathematics goes beyond the mundane. Mathematics enables the development of codes and ciphers that are useful to individuals and to society.

Learning outcomes

- Use coding schemes to encode and decode different types of information for identification, privacy, and security purposes.
- Exemplify honesty and integrity when using codes for security purposes.

International Standard Book Number

(International Standard Book Number, 2017)

https://en.wikipedia.org/wiki/File:ISBN_81-7525-766-0.svg

The ISBN-10 check digit must range from 0 to 10 (the symbol X is used for 10), and must be such that the sum of all the ten digits, each multiplied by its (integer) weight, descending from 10 to 1, is a multiple of 11.

The ISBN-13 check digit must range from 0 to 9 and must be such that the sum of all the thirteen digits, each multiplied by its (integer) weight, alternating between 1 and 3, is a multiple of 10.

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