Teaching Proportional Reasoning Concepts and Procedures Using Repetition with Variation

Topic Study Group 1: Quality Mathematics Curriculum and Materials

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Synopsis

We describe worksheets created for a study that aimed to provide empirical evidence on the causal relationships between conceptual and procedural knowledge in mathematics using an East Asian perspective.

The supplemental self-paced instructional materials are intended to help young adults improve their performance in tasks designed to assess their proportional reasoning understanding and skills.

The Worksheets

Students are to individually answer one written-response worksheet each day without using books or calculators.

They answer the worksheets at their own pace (taking around 15 to 30 minutes on the average to do so), prioritizing performance over speed.

Each worksheet is a booklet eight half-letter-sized pages long and includes a short discussion of the concepts or procedures involved, with examples and tasks arranged in a slowly increasing level of difficulty.

The tasks are to be done in a strictly sequential order and are to be repeated until mastery is attained.

Each of the eleven worksheets has a conceptual version (with nonnumeric tasks) and a procedural version (with numeric tasks).

A Locating numbers on a number line
B Identifying points on a number line
C Using a linear scale to represent ratios and proportions
D Comparing ratios without context
E Finding the mass of a liquid given its density and its volume
F Conserving linear speed (the product of a gear's number of teeth and angular speed)
G Dissolving grains in a liquid to get a solution with the same volume as the liquid
H Comparing the volumes of liquid in identical containers but with different orientations
I Decomposing and composing a solid with uniform density
J Conserving volume (the product of a liquid's height and area in a container)
K Review of worksheets D, E, F, G, H, I, and J

Locating numbers on a number line: Conceptual

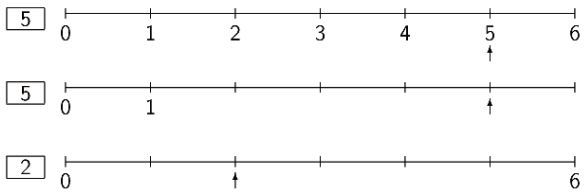
Aa1

1-20100329

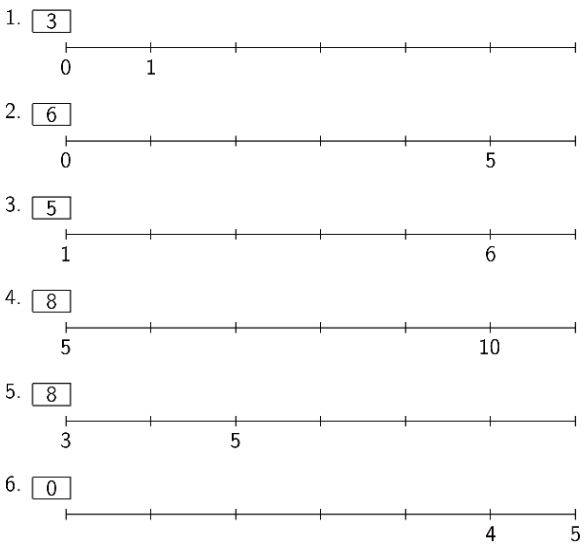
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

In each number line shown below, the arrow points to the given boxed number.

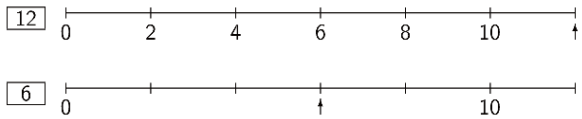


For each number line shown below, draw an arrow that points to the given number.

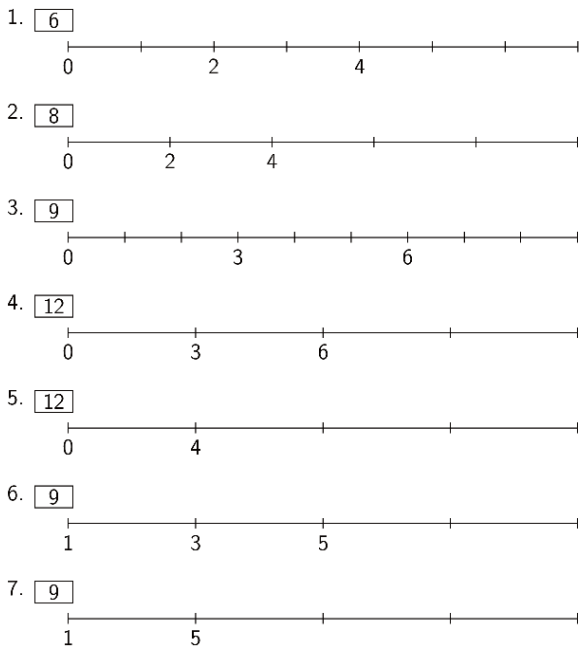


Aa2

In each number line shown below, the arrow points to the given boxed number.



For each number line shown below, draw an arrow that points to the given number.



Locating numbers on a number line: Conceptual

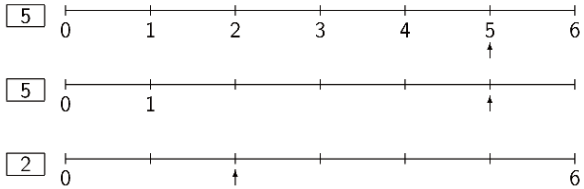
Aa1

conceptual (relationships)

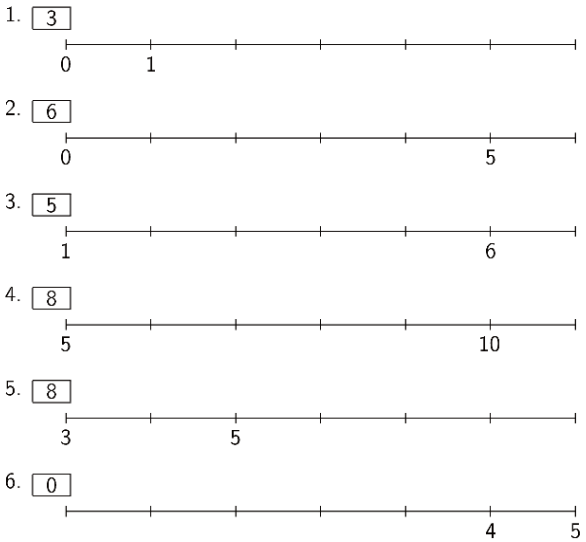
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

In each number line shown below, the arrow points to the given boxed number.

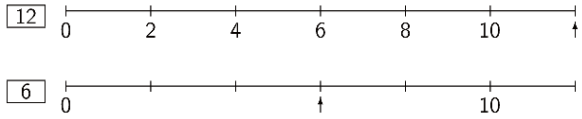


For each number line shown below, draw an arrow that points to the given number.



Aa2

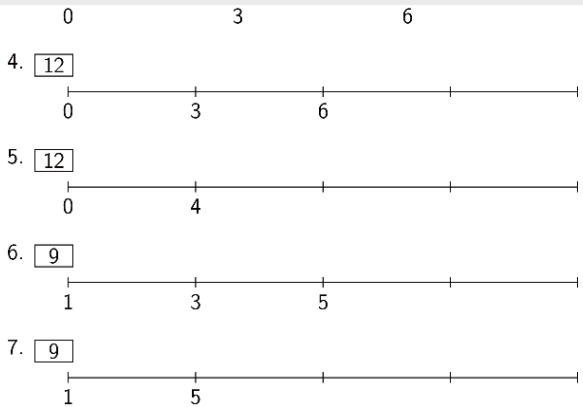
In each number line shown below, the arrow points to the given boxed number.



For each number line shown below, draw an arrow that points to the given number.

2. 8

nonnumeric tasks
(no arithmetic operations needed)



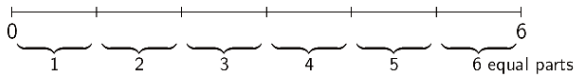
Locating numbers on a number line: Procedural

Ab1 5-20101105

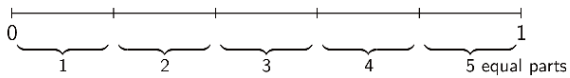
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

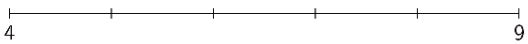
The number line shown below starts at the number 0 and ends at the number 6. It is divided into 6 equal parts.



The number line shown below starts at the number 0, ends at the number 1, and is divided into 5 equal parts.



1. Consider the number line shown below.

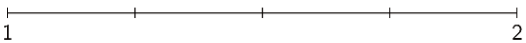


What number does it start with?

What number does it end with?

Into how many parts is it divided?

2. Consider the number line shown below.



What number does it start with?

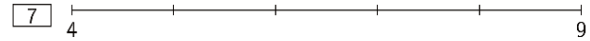
What number does it end with?

Into how many parts is it divided?

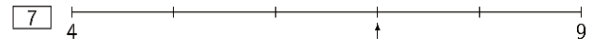
Ab2

Let a number line start at the number s, end at the number e, and be divided into p equal parts. An arrow that points to the end of the nth part from the left points to the number x, where $n = \frac{x-s}{e-s} \times p$.

For the number line shown below, $s = 4$, $e = 9$, $p = 5$, and an arrow that points to the given boxed number $x = 7$ is to be drawn.



The arrow is drawn pointing to the end of the nth part, where $n = \frac{x-s}{e-s} \times p = \frac{7-4}{9-4} \times 5 = \frac{3}{5} \times 5 = 3$. For this number line, an arrow pointing to the end of the third part points to the number 7.



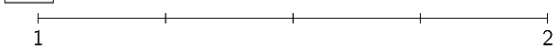
For each number line shown below, draw an arrow that points to the given boxed number. Do this by finding the value of n, then drawing an arrow that points to the end of the nth part.

1. $\boxed{3/5}$

$$n = \frac{x-s}{e-s} \times p = \frac{\frac{3}{5}-0}{1-0} \times 5 = \frac{\frac{3}{5}}{1} \times 5 = \frac{3}{5} \times 5 = 3$$

(Complete the arrow that points to the end of the third part.)

2. $\boxed{1\,3/4}$



$$n = \frac{1\frac{3}{4}-1}{2-1} \times 4 = \frac{3}{4} \times 4 = 3$$

(Draw an arrow that points to the end of the third part.)

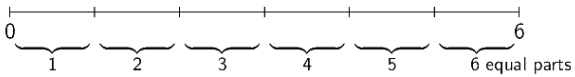
Locating numbers on a number line: Procedural

Ab1 procedural (techniques)

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

The number line shown below starts at the number 0 and ends at the number 6. It is divided into 6 equal parts.



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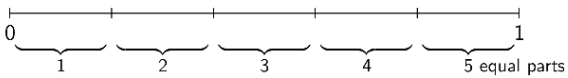
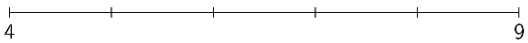


1. Consider the number line shown below.

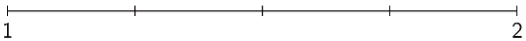


What number does it start with?

What number does it end with?

Into how many parts is it divided?

2. Consider the number line shown below.



What number does it start with?

What number does it end with?

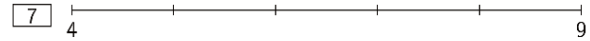
Into how many parts is it divided?

familiarity with symbols and syntax

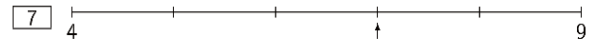
Ab2 rules or procedures

Let a number line start at the number s, end at the number e, and be divided into p equal parts. An arrow that points to the end of the nth part from the left points to the number x, where $n = \frac{x-s}{e-s} \times p$.

For the number line shown below, $s = 4$, $e = 9$, $p = 5$, and an arrow that points to the given boxed number $x = 7$ is to be drawn.



The arrow is drawn pointing to the end of the nth part, where $n = \frac{x-s}{e-s} \times p = \frac{7-4}{9-4} \times 5 = \frac{3}{5} \times 5 = 3$. For this number line, an arrow pointing to the end of the third part points to the number 7.



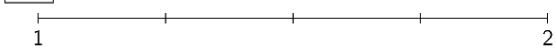
For each number line shown below, draw an arrow that points to the given boxed number. Do this by finding the value of n, then drawing an arrow that points to the end of the nth part.

1. $\boxed{3/5}$

$$n = \frac{x-s}{e-s} \times p = \frac{\frac{3}{5}-0}{1-0} \times 5 = \frac{\frac{3}{5}}{1} \times 5 = \frac{3}{5} \times 5 = 3$$

(Complete the arrow that points to the end of the third part.)

2. $\boxed{1\,3/4}$



$$n = \frac{1\frac{3}{4}-1}{2-1} \times 4 = \frac{3}{4} \times 4 = 3$$

numeric tasks (arithmetic operations needed)

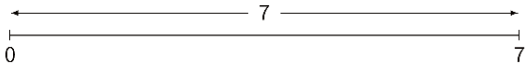
Identifying points on a number line: Conceptual

Ba1 4-20110104

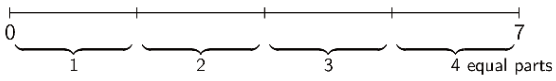
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

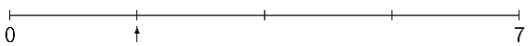
Let a number line have a length of 7 as shown below.



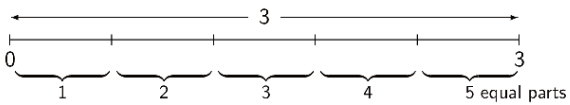
Let it be divided into 4 equal parts as shown below.



The figure shown below represents the division $7 \div 4$. The arrow points to the result of the operation $7 \div 4$, which can be represented as the fraction $^7/_4$.



The number line shown below has a length of 3 and is divided into 5 equal parts.



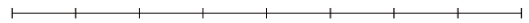
The figure shown below represents the division $3 \div 5$. The arrow points to the fraction $^3/_5$.

Ba2

1. Consider the figure shown below.

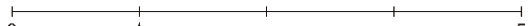
What division is represented? $8 \div \square$

What fraction does the arrow point to? $\frac{\square}{3}$

2. 

What division is represented? $\square \div 8$

What fraction does the arrow point to? $\frac{3}{\square}$

3. 

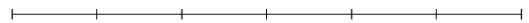
What division is represented? $\square \div \square$

What fraction does the arrow point to? $\frac{\square}{\square}$

4. 

What division is represented?

What fraction does the arrow point to?

5. 

What division is represented?

What fraction does the arrow point to?

6. 

What division is represented?

What fraction does the arrow point to?

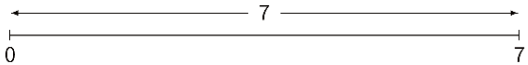
Identifying points on a number line: Conceptual

Ba1 4-20110104

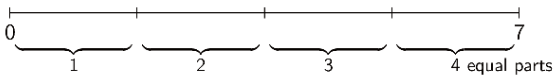
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

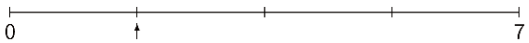
Let a number line have a length of 7 as shown below.



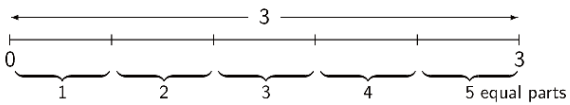
Let it be divided into 4 equal parts as shown below.



The figure shown below represents the division $7 \div 4$. The arrow points to the result of the operation $7 \div 4$, which can be represented as the fraction $^7/_4$.



The number line shown below has a length of 3 and is divided into 5 equal parts.



The figure shown below represents the division $3 \div 5$. The arrow points to the fraction $^3/_5$.

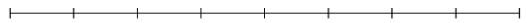
concept of a fraction as a quotient

Ba2

1. Consider the figure shown below.

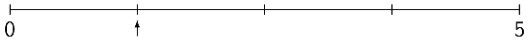
What division is represented? $8 \div \square$

What fraction does the arrow point to? $\frac{\square}{3}$

2. 

What division is represented? $\square \div 8$

What fraction does the arrow point to? $\frac{3}{\square}$

3. 

What division is represented? $\square \div \square$

What fraction does the arrow point to? $\frac{\square}{\square}$

4. 

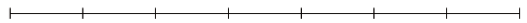
What division is represented?

What fraction does the arrow point to?

5. 

What division is represented?

What fraction does the arrow point to?

6. 

What division is represented?

What fraction does the arrow point to?

Identifying points on a number line: Procedural

Bb1 6-20110104

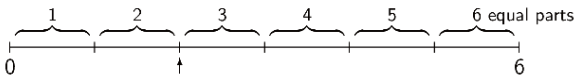
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

Let a number line start at the number s, end at the number e, and be divided into p equal parts. An arrow that points to the end of the nth part from the left points to the number $s + \frac{n}{p}(e - s)$.

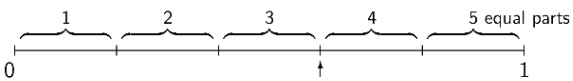
The number line shown below starts at the number $s = 0$ and ends at the number $e = 6$. It is divided into $p = 6$ equal parts. The arrow points to the end of the second part ($n = 2$).

The arrow points to the number $s+\frac{n}{p}(e-s) = 0+\frac{2}{6}(6-0) = 2$.



The number line shown below starts at $s = 0$, ends at $e = 1$, and is divided into $p = 5$ equal parts. The arrow points to the end of the third part ($n = 3$).

The arrow points to the number $s+\frac{n}{p}(e-s) = 0+\frac{3}{5}(1-0) = \frac{3}{5}$.



Find the number that the arrow points to.

1. 

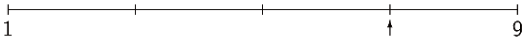
$$s + \frac{n}{p}(e - s) = 4 + \frac{3}{5}(9 - 4) = 4 + \frac{3}{\cancel{5}_1}(\cancel{5}^1) = 4 + 3 = \square$$

Bb2

For each number line shown, find the number that the arrow points to. Show your solutions.

1. 

$$s + \frac{n}{p}(e - s) = 1 + \frac{3}{5}(6 - 1) =$$

2. 

$$1 + \frac{3}{4}(9 - 1) =$$

3. 

Identifying points on a number line: Procedural

Bb1 6-20110104

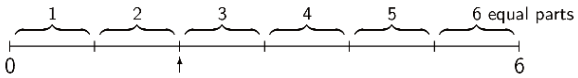
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

Let a number line start at the number s, end at the number e, and be divided into p equal parts. An arrow that points to the end of the nth part from the left points to the number $s + \frac{n}{p}(e - s)$.

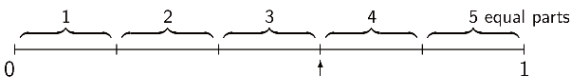
The number line shown below starts at the number $s = 0$ and ends at the number $e = 6$. It is divided into $p = 6$ equal parts. The arrow points to the end of the second part ($n = 2$).

The arrow points to the number $s+\frac{n}{p}(e-s) = 0+\frac{2}{6}(6-0) = 2$.



The number line shown below starts at $s = 0$, ends at $e = 1$, and is divided into $p = 5$ equal parts. The arrow points to the end of the third part ($n = 3$).

The arrow points to the number $s+\frac{n}{p}(e-s) = 0+\frac{3}{5}(1-0) = \frac{3}{5}$.



Find the number that the arrow points to.

1. 

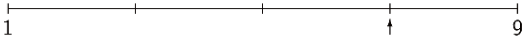
$$s + \frac{n}{p}(e - s) = 4 + \frac{3}{5}(9 - 4) = 4 + \frac{3}{\cancel{5}_1}(\cancel{5}^1) = 4 + 3 = \square$$

Bb2

For each number line shown, find the number that the arrow points to. Show your solutions.

1. 

$$s + \frac{n}{p}(e - s) = 1 + \frac{3}{5}(6 - 1) =$$

2. 

$$1 + \frac{3}{4}(9 - 1) =$$

3. 

procedure to identify a point on a number line

Ratio and proportion using a linear scale: Conceptual

Ca1 4-20110104

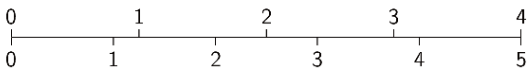
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

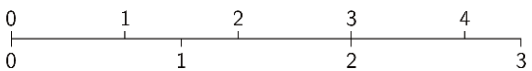
In the double-scale number line shown below, both scales start at zero, the upper scale is divided into five equal parts, the lower scale is divided into three equal parts, and the upper-scale number 5 and the lower-scale number 3 are at exactly the same point. This double-scale number line is described by the relationship 5:3 (pronounced "five is to three").

The double-scale number line shown below is described by the relationship 3:5. Note that the relationships 5:3 and 3:5 do not describe the same double-scale number line.

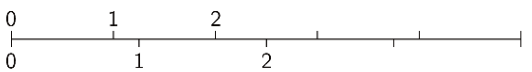
1. The double-scale number line shown below is described by the relationship 4:☐.



2. The number line shown below is described by 3:☐.

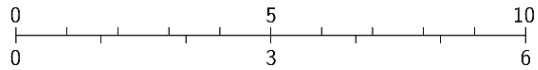


3. The number line shown below is described by 5:☐.



Ca2

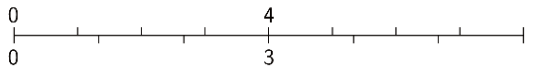
The double-scale number line shown below is described by the relationship 5:3 because the upper-scale 5 and the lower-scale 3 are at the same point. It is also described by the relationship 10:6 because the upper-scale 10 and the lower-scale 6 are at the same point. Thus, a double-scale number line can be described by more than one relationship.



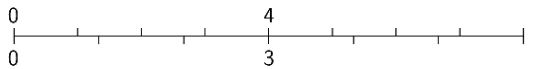
The statement $a : b :: c : d$ (pronounced "a is to b as c is to d") means that the relationships $a : b$ and $c : d$ describe the same double-scale number line. For the double-scale number line shown above, 5:3::10:6 ("five is to three as ten is to six"). It is also true that 10:6::5:3.

The double-scale number line shown below is described by the relationship 3:2. It is also described by the relationships 6:4 and 9:6.

1. The double-scale number line shown below is described by the relationship 8:☐.



2. For the number line shown below, 4:3::12:☐.



Ratio and proportion using a linear scale: Conceptual

Ca1 4-20110104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

In the double-scale number line shown below, both scales start at zero, the upper scale is divided into five equal parts, the lower scale is divided into three equal parts, and the upper-scale number 5 and the lower-scale number 3 are at exactly the same point. This double-scale number line is described by the relationship 5:3 (pronounced "five is to three").

The double-scale number line shown below is described by the relationship 3:5. Note that the relationships 5:3 and 3:5 do not describe the same double-scale number line.

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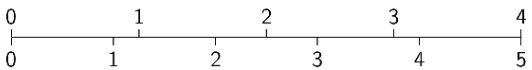

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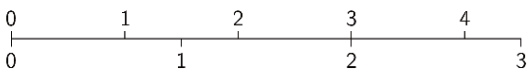

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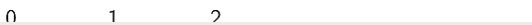
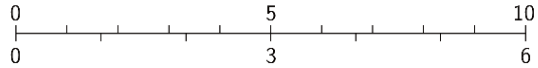


concept of a ratio as a relationship on a double-scale number line

Ca2

The double-scale number line shown below is described by the relationship 5:3 because the upper-scale 5 and the lower-scale 3 are at the same point. It is also described by the relationship 10:6 because the upper-scale 10 and the lower-scale 6 are at the same point. Thus, a double-scale number line can be described by more than one relationship.



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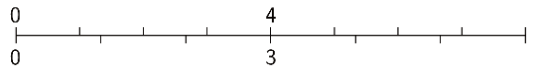

2. For the number line shown below, 4:3::12:☐.

concept of a proportion as two relationships on the same double-scale number line

Ratio and proportion using a linear scale: Procedural

Cb1 3-20110104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

In the double-scale number line shown below, both scales start at zero, the upper scale is divided into five equal parts, the lower scale is divided into three equal parts, and the upper-scale number 5 and the lower-scale number 3 are at exactly the same point. This double-scale number line is described by the relationship 5:3 (pronounced "five is to three").

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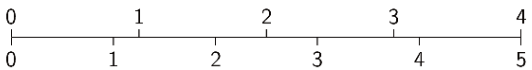

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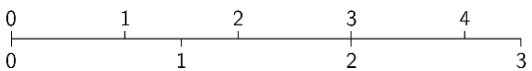

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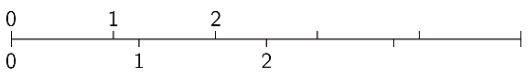

Cb2

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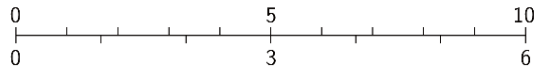

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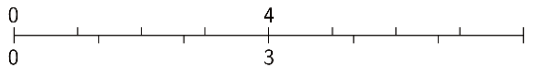

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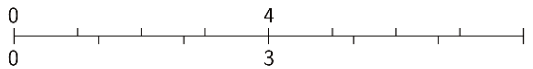

Ratio and proportion using a linear scale: Procedural

Cb1 3-20110104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

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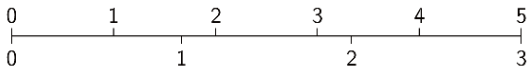

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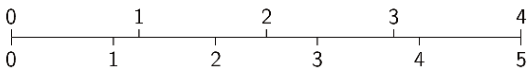
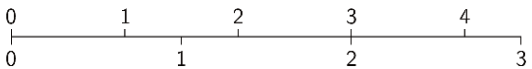


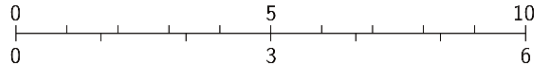
2. The number line shown below is described by 3:☐.



3. The number line shown below is described by 5:☐.

Cb2

The double-scale number line shown below is described by the relationship 5:3 because the upper-scale 5 and the lower-scale 3 are at the same point. It is also described by the relationship 10:6 because the upper-scale 10 and the lower-scale 6 are at the same point. Thus, a double-scale number line can be described by more than one relationship.



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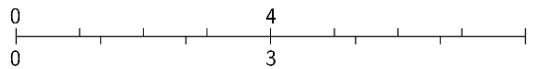

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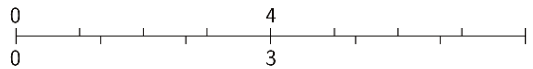

double-scale number line is new to students so conceptual and procedural versions have similar first two pages

Ratio comparison problems: Conceptual

Da1 2-20101105

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

Given two numbers located at different points on a number line, the one on the left is less than the one on the right, and the one on the right is greater than the one on the left. For example, in the number line shown below, $1/5$ is to the left of $3/5$, so $1/5$ is less than $3/5$, that is, $1/5 < 3/5$. Also, $3/5$ is to the right of $1/5$, so $3/5$ is greater than $1/5$, that is, $3/5 > 1/5$.

If two numbers are located at the same point on a number line, then they are equal. For example, $2/5$ is at the same point as $2/5$, so $2/5$ is equal to $2/5$, that is, $2/5 = 2/5$.

For each numbered item below, compare the values of the two fractions by writing $<$, $>$, or $=$ in the circled space.

1. $1/3$ ◯ $2/3$
2. $2/4$ ◯ $1/4$
3. $3/7$ ◯ $4/7$
4. $2/7$ ◯ $1/7$
5. $5/7$ ◯ $5/7$
6. $6/5$ ◯ $7/5$
7. $3/2$ ◯ $1/2$
8. $3/4$ ◯ $5/4$

Da2

The number lines below show that $3/7$ is to the left of $3/4$, so $3/7$ is less than $3/4$, that is, $3/7 < 3/4$. Also, $3/4$ is to the right of $3/7$, so $3/4$ is greater than $3/7$, that is, $3/4 > 3/7$.

For each item below, write $<$, $>$, or $=$ in the circled space.

1. $10/5$ ◯ $10/2$
2. $6/3$ ◯ $6/2$
3. $8/2$ ◯ $8/4$
4. $1/7$ ◯ $1/2$
5. $1/3$ ◯ $1/4$
6. $2/4$ ◯ $2/3$
7. $3/4$ ◯ $3/5$
8. $4/8$ ◯ $4/7$
9. $4/5$ ◯ $4/3$
10. $5/2$ ◯ $5/3$

Ratio comparison problems: Conceptual

Da1 2-20101105

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

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3. $3/7$ ◯ $4/7$
4. $2/7$ ◯ $1/7$
5. $5/7$ ◯ $5/7$
6. $6/5$ ◯ $7/5$
7. $3/2$ ◯ $1/2$
8. $3/4$ ◯ $5/4$

comparing fractions with common denominators (no arithmetic operations needed)

Da2

The number lines below show that $3/7$ is to the left of $3/4$, so $3/7$ is less than $3/4$, that is, $3/7 < 3/4$. Also, $3/4$ is to the right of $3/7$, so $3/4$ is greater than $3/7$, that is, $3/4 > 3/7$.

For each item below, write $<$, $>$, or $=$ in the circled space.

1. $10/5$ ◯ $10/2$
2. $6/3$ ◯ $6/2$
3. $8/2$ ◯ $8/4$
4. $1/7$ ◯ $1/2$
5. $1/3$ ◯ $1/4$
6. $2/4$ ◯ $2/3$
7. $3/4$ ◯ $3/5$
8. $4/8$ ◯ $4/7$
9. $4/5$ ◯ $4/3$
10. $5/2$ ◯ $5/3$

fractions as points on a number line

common numerators

Ratio comparison problems: Procedural

Db1 3-20101105

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

The relationship between two fractions $^a/_c$ and $^b/_d$ (where a, b, c, and d are all positive) is the same as the relationship between the products ad and bc.

To find the relationship between $^a/_c = {}^1/_2$ and $^b/_d = {}^3/_4$, note that $ad = 1{\times}4 = 4$, $bc = 3{\times}2 = 6$, $ad < bc$ (because $4 < 6$) and so $^a/_c < {}^b/_d$, that is, $^1/_2 < {}^3/_4$.

To find the relationship between $^3/_5$ and $^4/_7$, note that $3{\times}7 > 4{\times}5$ (because $21 > 20$) and so $^3/_5 > {}^4/_7$.

To find the relationship between $^4/_6$ and $^6/_9$, note that $4{\times}9 = 6{\times}6$ (because $36 = 36$) and so $^4/_6 = {}^6/_9$.

For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

1. $^1/_2$ ◯ $^3/_4$ $\quad 1 \times 4 \,?\, 3 \times 2$, $4 < 6$

2. $^3/_5$ ◯ $^4/_7$ $\quad 3 \times 7 \,?\, 4 \times 5$, $21 \,◯\, 20$

3. $^4/_6$ ◯ $^6/_9$ $\quad 4 \times 9 \,?\, 6 \times 6$

Db2

For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

1. $^1/_4$ ◯ $^2/_4$ $\quad 1 \times 4 \,?\, 2 \times 4$, $4 < 8$

2. $^3/_4$ ◯ $^2/_4$ $\quad 3 \times 4 \,?\, 2 \times 4$

3. $^1/_2$ ◯ $^1/_3$

4. $^1/_4$ ◯ $^1/_3$

5. $^2/_4$ ◯ $^2/_3$

6. $^2/_4$ ◯ $^2/_5$

Ratio comparison problems: Procedural

Db1 3-20101105

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

The relationship between two fractions $^a/_c$ and $^b/_d$ (where a, b, c, and d are all positive) is the same as the relationship between the products ad and bc.

To find the relationship between $^a/_c = {}^1/_2$ and $^b/_d = {}^3/_4$, note that $ad = 1{\times}4 = 4$, $bc = 3{\times}2 = 6$, $ad < bc$ (because $4 < 6$) and so $^a/_c < {}^b/_d$, that is, $^1/_2 < {}^3/_4$.

To find the relationship between $^3/_5$ and $^4/_7$, note that $3{\times}7 > 4{\times}5$ (because $21 > 20$) and so $^3/_5 > {}^4/_7$.

To find the relationship between $^4/_6$ and $^6/_9$, note that $4{\times}9 = 6{\times}6$ (because $36 = 36$) and so $^4/_6 = {}^6/_9$.

For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

1. $^1/_2$ ◯ $^3/_4$ $\quad 1 \times 4 \,⃝?\, 3 \times 2$, $4 < 6$

2. $^3/_5$ ◯ $^4/_7$ $\quad 3 \times 7 \,⃝?\, 4 \times 5$, $21 \,◯\, 20$

3. $^4/_6$ ◯ $^6/_9$ $\quad 4 \times 9 \,⃝?\, 6 \times 6$

comparing fractions (arithmetic operations needed)

Db2

For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

1. $^1/_4$ ◯ $^2/_4$ $\quad 1 \times 4 \,⃝?\, 2 \times 4$, $4 < 8$

2. $^3/_4$ ◯ $^2/_4$ $\quad 3 \times 4 \,⃝?\, 2 \times 4$

3. $^1/_2$ ◯ $^1/_3$

4. $^1/_4$ ◯ $^1/_3$

5. $^2/_4$ ◯ $^2/_3$

6. $^2/_4$ ◯ $^2/_5$

to emphasize procedure, student is asked to show calculations

Mass of a liquid: Conceptual

Ea1 3-20101104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

1. One liter (1 L) of a certain liquid weighs 1 kilogram (1 kg). When full, the container shown at the right holds 1 L of this liquid. How many kilograms does the liquid shown at the right weigh?

   ☐ kg

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L. How many kilograms does the liquid shown at the right weigh?

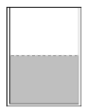
4. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?

5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?

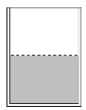
Ea2

1. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 1 L. How many kilograms does the liquid shown at the right weigh?

   ☐ kg



2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?



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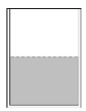
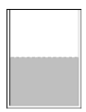
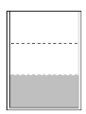


4. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?



5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



Mass of a liquid: Conceptual

Ea1

3-20101104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

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   ☐ kg

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?

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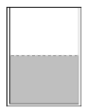
5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?

no procedure given

Ea2

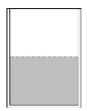
1. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 1 L. How many kilograms does the liquid shown at the right weigh?



   $\frac{\square}{\square}$ kg

2. 1 L of a cert
   the container
   many kilogram
   weigh?

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L. How many kilograms does the liquid shown at the right weigh?



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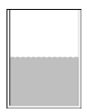
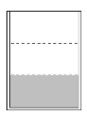


5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



conservation of density

hints on expected form of answer (fraction, not decimal)

concepts taught through repetition with variation, highly sequential presentation

Mass of a liquid: Procedural

Eb1 4-20110106

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

If v_1 liters of a certain liquid weigh m_1 kilograms, then v_2 liters weigh $m_2 = \frac{m_1}{v_1} v_2$.

For each numbered item below, find the weight in kilograms of the liquid shown at the right. Show your solutions.

1. One liter (1 L) of a certain liquid weighs 1 kilogram (1 kg). When full, the container shown at the right holds 1 L of this liquid.

$$m_2 = \frac{m_1}{v_1} v_2 = \frac{1 \text{ kg}}{1 \text{ L}} \cdot 1 \text{ L} = \square \text{ kg}$$

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.

$$m_2 = \frac{1 \text{ kg}}{1 \text{ L}} \cdot 2 \text{ L} =$$

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L.

$$\frac{2 \text{ kg}}{1 \text{ L}} \cdot 1 \text{ L} =$$

4. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 2 L.

$$\frac{2 \text{ kg}}{1 \text{ L}} \cdot 2 \text{ L} =$$

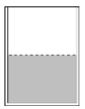
5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.

$$\frac{2 \text{ kg}}{1 \text{ L}} \cdot 3 \text{ L} =$$

Eb2

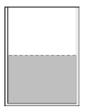
Find the weight in kilograms of the liquid shown at the right given the following conditions. Show your solutions.

1. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 1 L.



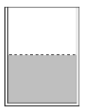
$$\frac{1 \text{ kg}}{1 \text{ L}} \cdot \frac{1}{2}(1 \text{ L}) = \frac{\square}{\square} \text{ kg}$$

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.



$$\frac{1 \text{ kg}}{1 \text{ L}} \cdot \frac{1}{2}(2 \text{ L}) = \square \text{ kg}$$

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L.



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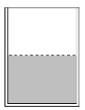
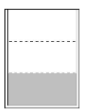


5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.



Mass of a liquid: Procedural

Eb1 4-20110106

Student number: ☐☐☐☐-☐

Date: _____

procedure given and illustrated

If v_1 liters of a certain liquid weigh m_1 kilograms, then v_2 liters weigh $m_2 = \frac{m_1}{v_1} v_2$.

For each numbered item below, find the weight in kilograms of the liquid shown at the right. Show your solutions.

1. One liter (1 L) of a certain liquid weighs 1 kilogram (1 kg). When full, the container shown at the right holds 1 L of this liquid.

$$m_2 = \frac{m_1}{v_1} v_2 = \frac{1 \text{ kg}}{1 \text{ ℓ}} \cdot 1 \text{ ℓ} = \square \text{ kg}$$

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.

$$m_2 = \frac{1 \text{ kg}}{1 \text{ ℓ}} \cdot 2 \text{ ℓ} =$$

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L.

$$\frac{2 \text{ kg}}{1 \text{ ℓ}} \cdot 1 \text{ ℓ} =$$

4. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 2 L.

$$\frac{2 \text{ kg}}{1 \text{ ℓ}} \cdot 2 \text{ ℓ} =$$

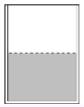
5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.

$$\frac{2 \text{ kg}}{1 \text{ ℓ}} \cdot 3 \text{ ℓ} =$$

Eb2

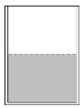
Find the weight in kilograms of the liquid shown at the right given the following conditions. Show your solutions.

1. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 1 L.



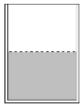
$$\frac{1 \text{ kg}}{1 \text{ ℓ}} \cdot \frac{1}{2}(1 \text{ ℓ}) = \frac{\square}{\square} \text{ kg}$$

2. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.



$$\frac{1 \text{ kg}}{1 \text{ ℓ}} \cdot \frac{1}{2}(2 \text{ ℓ}) = \square \text{ kg}$$

3. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 1 L.



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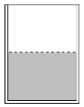
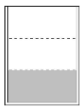


5. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.



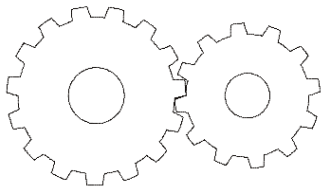
proportion as an equality of two rates (density is the quotient of mass and volume)

Interlocking toothed gears: Conceptual

Fa1 4-20110106

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____



Two interlocking toothed gears are shown above. Each gear can rotate around an imaginary line through its center called its *axis*. If the gear rotates in the same direction as the hand of a working clock, it is said to be rotating *clockwise*. If it is rotating in the opposite direction, it is said to be rotating *counter-clockwise*. The rate of rotation of the gear around its axis is called its *angular speed*.

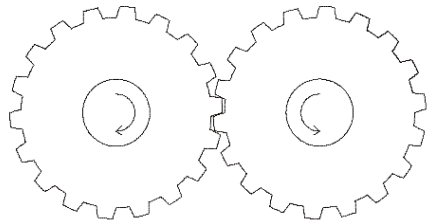
If two gears are interlocked, then they rotate in opposite directions.

If two interlocked gears have the same number of teeth, then they rotate at the same speed. If one gear has fewer teeth than the other, then it would rotate faster than the other.

For example, if the leftmost gear was rotating clockwise, then the rightmost gear would be rotating counter-clockwise. The rightmost gear has fewer teeth than the leftmost gear, so the rightmost gear would rotate faster than the leftmost gear.

NOTE: Assume that all the gears shown in this set of worksheets interlock correctly and do not jam or slip.

Fa2



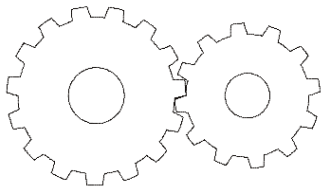
1. The leftmost gear is rotating clockwise, as shown by the curved arrow. What can be said about the rightmost gear?
   (a) It is rotating clockwise.
   (b) It is rotating counter-clockwise.
   (c) Its direction of rotation cannot be determined.
2. Which of the following is true?
   (a) The leftmost gear has more teeth.
   (b) The rightmost gear has more teeth.
   (c) The two gears have the same number of teeth.
3. What can be said about the angular speeds of the two gears?
   (a) The leftmost gear is rotating faster.
   (b) The rightmost gear is rotating faster.
   (c) Both gears are rotating at the same angular speed.
   (d) It cannot be determined which gear is rotating faster.

Interlocking toothed gears: Conceptual

Fa1 4-20110106

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____



Two interlocking toothed gears are shown above. Each gear can rotate around an imaginary line through its center called its *axis*. If the gear rotates in the same direction as the hand of a working clock, it is said to be rotating *clockwise*. If it is rotating in the opposite direction, it is said to be rotating *counter-clockwise*. The rate of rotation of the gear around its axis is called its *angular speed*.

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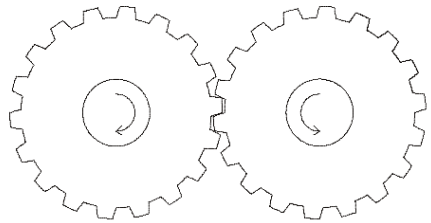
NOTE: Assume that all the gears shown in this set of worksheets interlock correctly and do not jam or slip.

Fa2



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   (a) The leftmost gear is rotating faster.
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   (c) Both gears are rotating at the same angular speed.
   (d) It cannot be determined which gear is rotating faster.

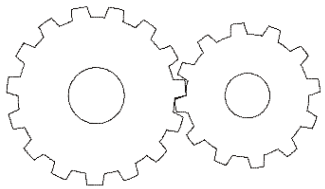
conservation of linear speed

Interlocking toothed gears: Procedural

Fb1 4-20101104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____



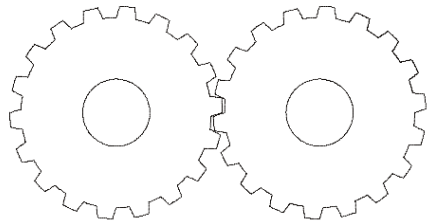
Two interlocking toothed gears are shown above. Each gear can rotate around an imaginary line through its center called its *axis*. The rate of rotation of the gear around its axis is called its *angular speed*.

Given two interlocked gears (labeled gear 1 and gear 2), let n_1 be the number of teeth of gear 1 and n_2 be the number of teeth of gear 2. Let ω_1 and ω_2 be the angular speeds of gears 1 and 2, respectively. The relationship between the number of teeth and the angular speeds of two interlocked gears are given by the equation $n_1\omega_1 = n_2\omega_2$.

For example, the leftmost gear (call it gear 1) has 15 teeth and the rightmost gear (gear 2) has 12 teeth. If, say, gear 1 is rotating at 4 revolutions per second, then gear 2 is rotating at an angular speed of $\omega_2 = n_1\omega_1/n_2 = (15 \times 4 \text{ rev/sec})/12 = 5 \text{ rev/sec}$.

NOTE: Assume that all the gears shown in this set of worksheets interlock correctly and do not jam or slip.

Fb2



1. How many teeth does the leftmost gear have? $n_1 =$
2. How many teeth does the rightmost gear have? $n_2 =$
3. The leftmost gear is rotating at $\omega_1 = 3$ rev/sec. What is the angular speed of the rightmost gear (in rev/sec)? Show your solutions.

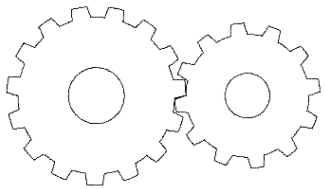
   $\omega_2 = n_1\omega_1/n_2 = (18 \times 3 \text{ rev/scc})/18 = \square \text{ rev/scc}$

Interlocking toothed gears: Procedural

Fb1 4-20101104

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____



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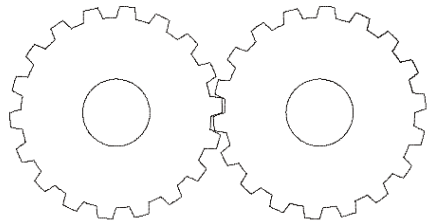
Given two interlocked gears (labeled gear 1 and gear 2), let n_1 be the number of teeth of gear 1 and n_2 be the number of teeth of gear 2. Let ω_1 and ω_2 be the angular speeds of gears 1 and 2, respectively. The relationship between the number of teeth and the angular speeds of two interlocked gears are given by the equation $n_1\omega_1 = n_2\omega_2$.

For example, the leftmost gear (call it gear 1) has 15 teeth and the rightmost gear (gear 2) has 12 teeth. If, say, gear 1 is rotating at 4 revolutions per second, then gear 2 is rotating at an angular speed of $\omega_2 = n_1\omega_1/n_2 = (15 \times 4 \text{ rev/sec})/12 = 5 \text{ rev/sec}$.

NOTE: Assume that all the gears shown in this set of worksheets interlock correctly and do not jam or slip.

Fb2



1. How many teeth does the leftmost gear have? $n_1 =$
2. How many teeth does the rightmost gear have? $n_2 =$
3. The leftmost gear is rotating at $\omega_1 = 3$ rev/sec. What is the angular speed of the rightmost gear (in rev/sec)? Show your solutions.

   $\omega_2 = n_1\omega_1/n_2 = (18 \times 3 \text{ rev/scc})/18 = \square \text{ rev/scc}$

proportion as an equality of two products of measure
(linear speed is the product of a gear's number of teeth and angular speed)

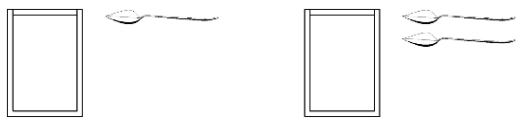
Sugar and water: Conceptual

Ga1 3-20110111

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

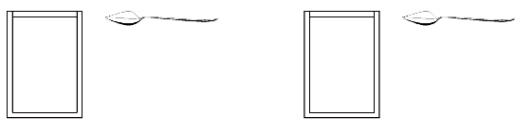
Amounts of sugar will be mixed thoroughly with amounts of water in two identical containers. Each resulting sugar and water solution will have a corresponding level of sweetness.

Shown below are two containers. The one on the left is full of water to be mixed with one spoonful of sugar. The one on the right is full of water to be mixed with two spoonfuls of sugar. The solution on the right will be sweeter.



Shown below are two containers. The one on the left is half-full of water to be mixed with one spoonful of sugar. The one on the right is full of water to be mixed with one spoonful of sugar. The solution on the left will be sweeter.

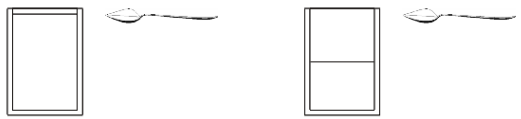
1. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

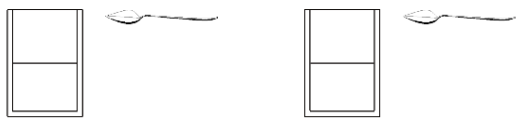
Ga2

1. What can be said about the two solutions shown below?



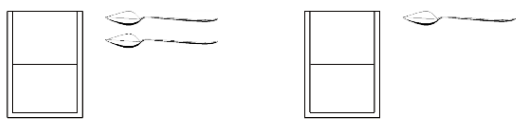
(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

2. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

3. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

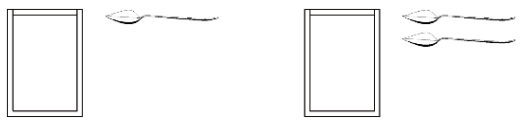
Sugar and water: Conceptual

Ga1 3-20110111

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

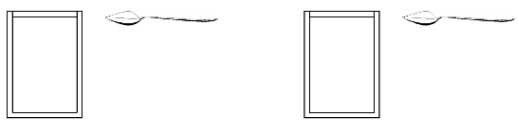
Amounts of sugar will be mixed thoroughly with amounts of water in two identical containers. Each resulting sugar and water solution will have a corresponding level of sweetness.

Shown below are two containers. The one on the left is full of water to be mixed with one spoonful of sugar. The one on the right is full of water to be mixed with two spoonfuls of sugar. The solution on the right will be sweeter.



Shown below are two containers. The one on the left is half-full of water to be mixed with one spoonful of sugar. The one on the right is full of water to be mixed with one spoonful of sugar. The solution on the left will be sweeter.

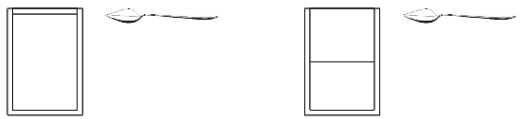
1. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

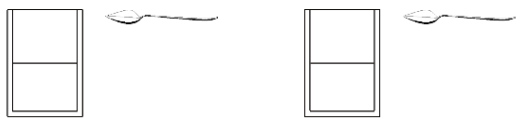
Ga2

1. What can be said about the two solutions shown below?



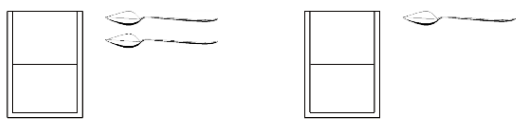
(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

2. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

3. What can be said about the two solutions shown below?



(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) The two solutions will have the same sweetness.

dissolving grains in a liquid to get a solution with the same volume as the liquid

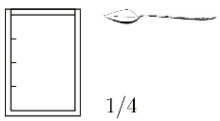
Sugar and water: Procedural

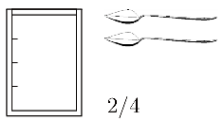
Gb1 2-20110111

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

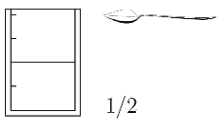
Sugar will be mixed thoroughly with water in two identical containers. Each resulting solution will have a level of sweetness.

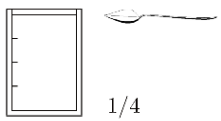
The container on the left below has 1 spoonful of sugar and 4 units of water. Its sweetness will be $1/4$ spoonful of sugar per unit of water. The one on the right below has 2 spoonfuls of sugar and 4 units of water. Its sweetness will be $2/4$ spoonful of sugar per unit of water. The solution on the right will be sweeter because $2/4 > 1/4$.



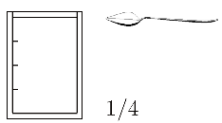


The container on the left below will have a sweetness of $1/2$ spoonful of sugar per unit of water. The one on the right below will have a sweetness of $1/4$ spoonful of sugar per unit of water. The solution on the left will be sweeter because $1/2 > 1/4$.

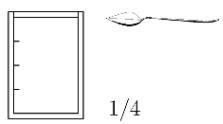




1. The solutions below will have the indicated sweetness (in spoonful of sugar per unit of water). Which will be sweeter?



(a) The one on the left
(b) The one on the right

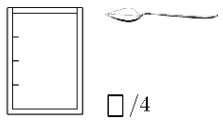


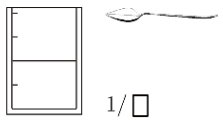
(c) Neither. The two will have the same sweetness.

Gb2

Indicate the sweetness (in spoonful of sugar per unit of water) a solution will have by writing the number of spoonfuls of sugar in the numerator and the number of units of water in the denominator. The solution with the larger fraction will be sweeter.

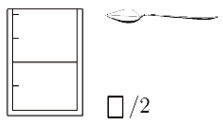
1. Indicate the sweetness of each solution. Which will be sweeter?

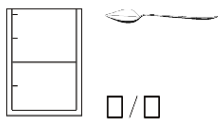




(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

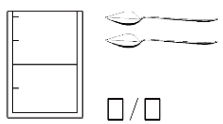
2. Indicate the sweetness of each solution. Which will be sweeter?

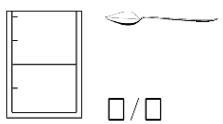




(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

3. Indicate the sweetness of each solution. Which will be sweeter?





(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

Sugar and water: Procedural

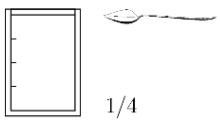
procedure: express sweetness as a fraction

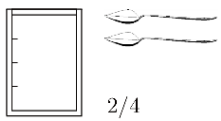
Gb1 2-20110111

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

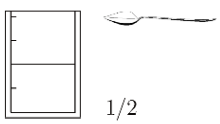
Sugar will be mixed thoroughly with water in two identical containers. Each resulting solution will have a level of sweetness.

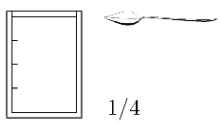
The container on the left below has 1 spoonful of sugar and 4 units of water. Its sweetness will be $1/4$ spoonful of sugar per unit of water. The one on the right below has 2 spoonfuls of sugar and 4 units of water. Its sweetness will be $2/4$ spoonful of sugar per unit of water. The solution on the right will be sweeter because $2/4 > 1/4$.



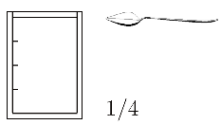


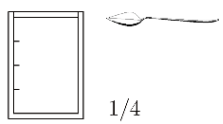
The container on the left below will have a sweetness of $1/2$ spoonful of sugar per unit of water. The one on the right below will have a sweetness of $1/4$ spoonful of sugar per unit of water. The solution on the left will be sweeter because $1/2 > 1/4$.





1. The solutions below will have the indicated sweetness (in spoonful of sugar per unit of water). Which will be sweeter?

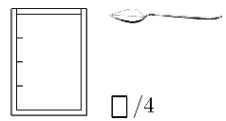


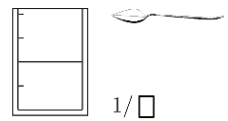


(a) The one on the left
(b) The one on the right
(c) Neither. The two will have the same sweetness.

Indicate the sweetness (in spoonful of sugar per unit of water) a solution will have by writing the number of spoonfuls of sugar in the numerator and the number of units of water in the denominator. The solution with the larger fraction will be sweeter.

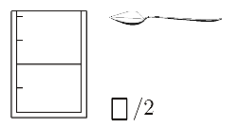
1. Indicate the sweetness of each solution. Which will be sweeter?

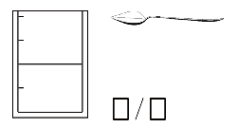




(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

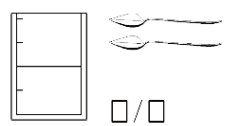
2. Indicate the sweetness of each solution. Which will be sweeter?

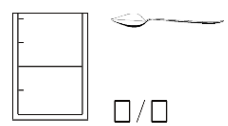




(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

3. Indicate the sweetness of each solution. Which will be sweeter?





(a) The solution on the left will be sweeter.
(b) The solution on the right will be sweeter.
(c) Neither. The two will have the same sweetness.

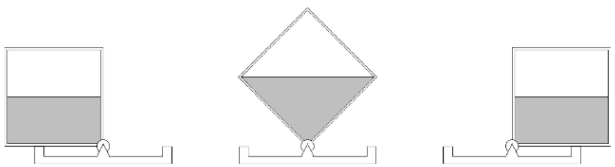
Water rectangle: Conceptual

Ha1 2-20100722

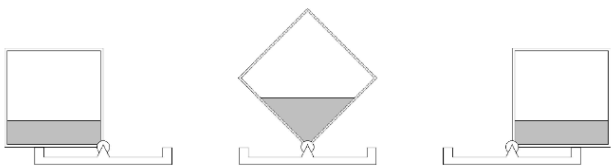
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

Colored liquid is put into rectangular containers with transparent sides. The containers are then sealed. The containers have uniform thickness and can be tilted from one resting position to another.

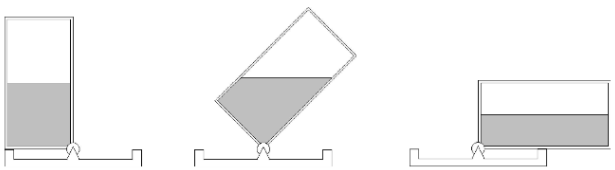
One container is shown being tilted below. It is shown below left resting on one side, below center being tilted to the other side, and below right resting on the other side.



The drawings below show a different container, identical in size and shape to the one above. Note that this container contains less liquid than the one above.



The drawings below show a different container, differing in size and shape from the two containers shown above.

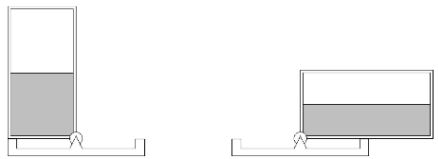


Ha2

Let there be a pair of identical rectangular containers. Colored liquid is put into each container, then the containers are sealed. The amount of liquid put into the containers may differ or may be the same.

Shown below is a pair of identical containers. On the left is one container lying on one side; on the right is the other container lying on its other side. The container on the left has more liquid than the one on the right.

A different pair of identical containers is shown below. The two containers have the same amount of liquid.



1. Which of the two containers shown below has more liquid?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Water rectangle: Conceptual

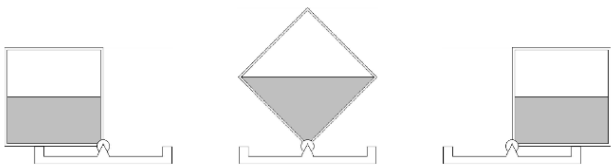
Ha1 2-20100722

novel task: same container in different orientations

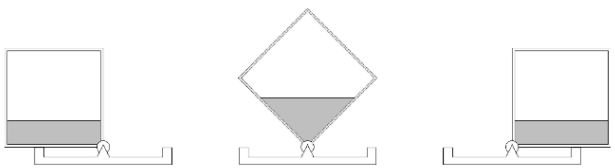
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

Colored liquid is put into rectangular containers with transparent sides. The containers are then sealed. The containers have uniform thickness and can be tilted from one resting position to another.

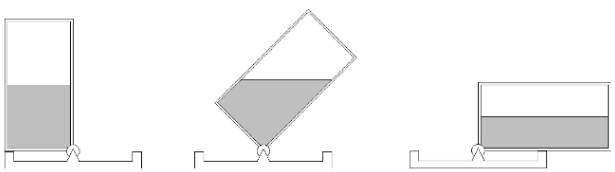
One container is shown being tilted below. It is shown below left resting on one side, below center being tilted to the other side, and below right resting on the other side.



The drawings below show a different container, identical in size and shape to the one above. Note that this container contains less liquid than the one above.



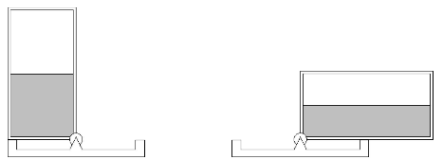
The drawings below show a different container, differing in size and shape from the two containers shown above.



Let there be a pair of identical rectangular containers. Colored liquid is put into each container, then the containers are sealed. The amount of liquid put into the containers may differ or may be the same.

Shown below is a pair of identical containers. On the left is one container lying on one side; on the right is the other container lying on its other side. The container on the left has more liquid than the one on the right.

A different pair of identical containers is shown below. The two containers have the same amount of liquid.



1. Which of the two containers shown below has more liquid?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

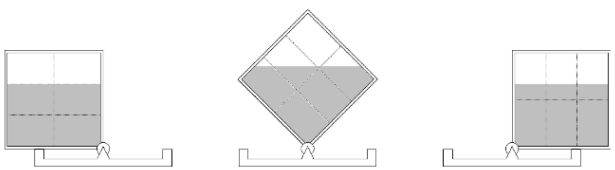
Water rectangle: Procedural

Hb1 2-20110111

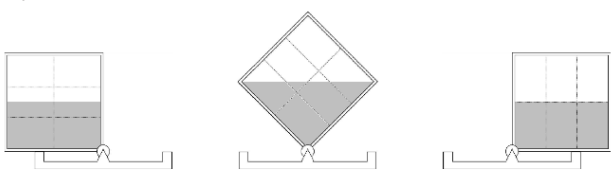
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

Colored liquid is put into rectangular containers with transparent sides marked with a rectangular grid. The containers are then sealed. The containers have uniform thickness and can be tilted from one resting position to another.

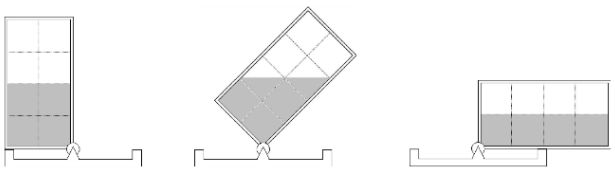
One container is shown being tilted below. It is shown below left resting on one side, below center being tilted to the other side, and below right resting on the other side.



The drawings below show a different container, identical in size and shape to the one above. Note that this container contains less liquid than the one above.



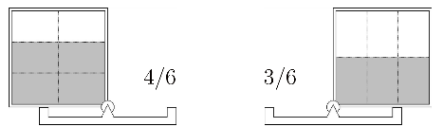
The drawings below show a different container, differing in size and shape from the two containers shown above.



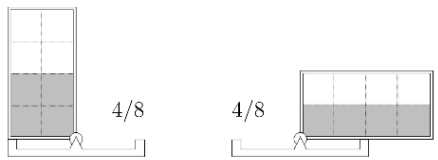
Hb2

Let there be a pair of identical rectangular containers. Colored liquid is put into each container, then the containers are sealed. The amount of liquid put into the containers may differ or may be the same.

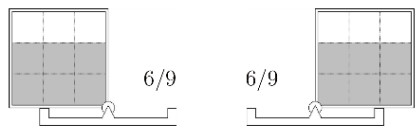
Shown below is a pair of identical containers. On the left is one container lying on one side; on the right is the other container lying on its other side. The container on the left has more liquid than the one on the right because the liquid on the left covers a larger fraction of the same grid (4/6) than that on the right (3/6).



A different pair of identical containers is shown below. The two containers have the same amount of liquid because in both the liquid covers the same fraction of the same grid (4/8).



1. Which of the two containers shown below has more liquid?



(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Neither. Both have the same amount of liquid.

Water rectangle: Procedural

rectangular grid allows numeric comparison

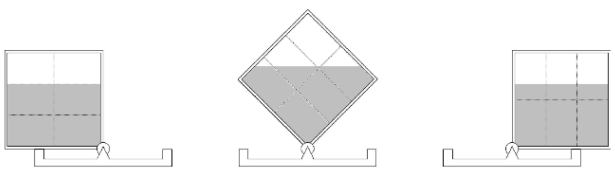
Hb1 2-20110111

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

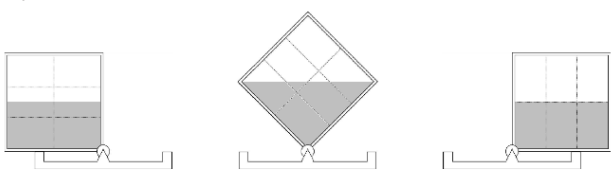
Date: _____ Time finished: _____

Colored liquid is put into rectangular containers with transparent sides marked with a rectangular grid. The containers are then sealed. The containers have uniform thickness and can be tilted from one resting position to another.

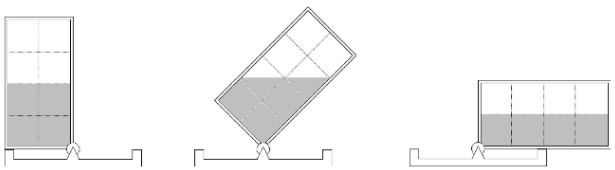
One container is shown being tilted below. It is shown below left resting on one side, below center being tilted to the other side, and below right resting on the other side.



The drawings below show a different container, identical in size and shape to the one above. Note that this container contains less liquid than the one above.



The drawings below show a different container, differing in size and shape from the two containers shown above.



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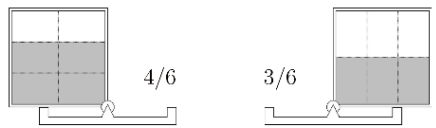
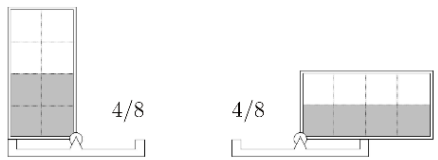
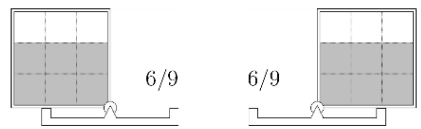
Shown below is a pair of identical containers. On the left is one container lying on one side; on the right is the other container lying on its other side. The container on the left has more liquid than the one on the right because the liquid on the left covers a larger fraction of the same grid (4/6) than that on the right (3/6).



A different pair of identical containers is shown below. The two containers have the same amount of liquid because in both the liquid covers the same fraction of the same grid (4/8).



1. Which of the two containers shown below has more liquid?



(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Neither. Both have the same amount of liquid.

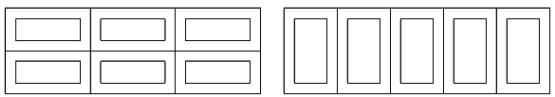
Masses of chocolate bar pieces: Conceptual

Ia1 3-20110113

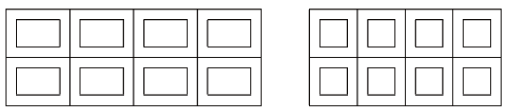
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

Let chocolate bars be composed of identical smaller pieces. The smaller pieces of a given chocolate bar have the same size, shape, and mass. Pieces from different chocolate bars may differ in size, shape, and mass.

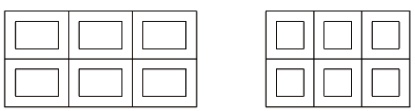
Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. The chocolate bar on the left has more pieces than the one on the right. Thus, the chocolate bar on the left has more mass than the one on the right.



Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. The two chocolate bars have the same number of smaller pieces. Thus, the two chocolate bars have the same mass.



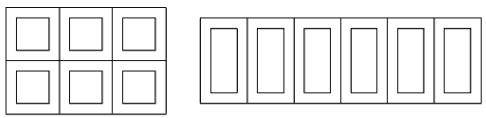
1. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

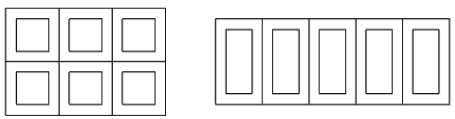
Ia2

1. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



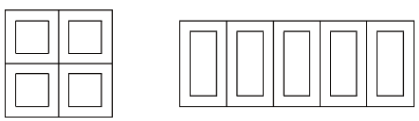
(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

2. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

3. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

Masses of chocolate bar pieces: Conceptual

decomposition and composition of a solid with uniform density

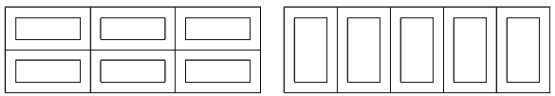
Ia1 3-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

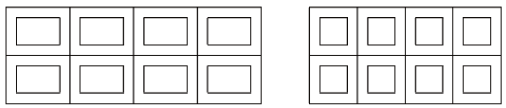
Date: _____ Time finished: _____

Let chocolate bars be composed of identical smaller pieces. The smaller pieces of a given chocolate bar have the same size, shape, and mass. Pieces from different chocolate bars may differ in size, shape, and mass.

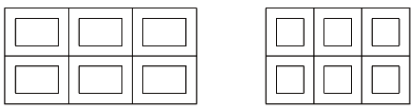
Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. The chocolate bar on the left has more pieces than the one on the right. Thus, the chocolate bar on the left has more mass than the one on the right.



Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. The two chocolate bars have the same number of smaller pieces. Thus, the two chocolate bars have the same mass.

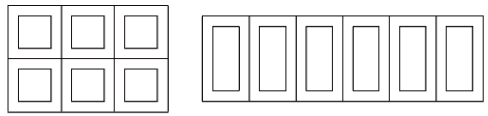


1. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



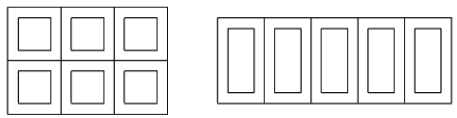
(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

1. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



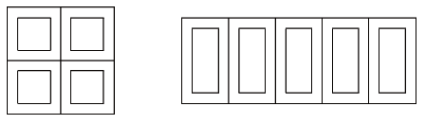
(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

2. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

3. Each smaller piece of the chocolate bar shown below left has the same mass as each smaller piece of the chocolate bar shown below right. Which chocolate bar has more mass?



(a) The one on the left has more mass.
(b) The one on the right has more mass.
(c) They both have the same mass.

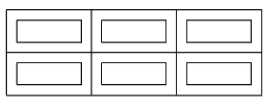
Masses of chocolate bar pieces: Procedural

Ib1 4-20110113

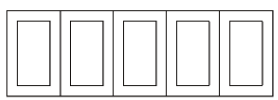
Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

Let chocolate bars be composed of identical smaller pieces. The smaller pieces of a given chocolate bar have the same size, shape, and mass. Pieces from different chocolate bars may differ in size, shape, and mass.

The chocolate bar shown below has a mass of 60 grams. It is composed of 6 smaller pieces. Each piece has a mass of 60 g ÷ 6 = 10 g.

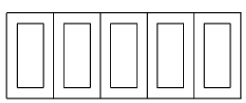


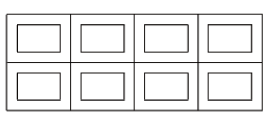
The chocolate bar shown below has a mass of 100 g. It is composed of 5 smaller pieces. Each piece has a mass of 100 g ÷ 5 = 20 g.



In general, if a chocolate bar has a mass of m and is composed of n smaller pieces, then each piece has a mass of m/n.

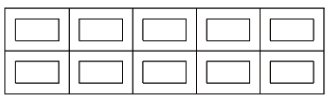
For each chocolate bar shown, find the mass of each smaller piece. Show your solutions.

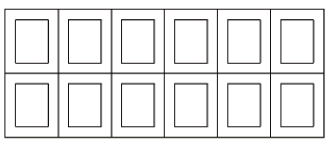
1.  $m = 20$ g
$\frac{m}{n} = \frac{20 \text{ g}}{5} = \square$ g

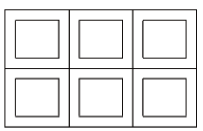
2.  $m = 40$ g
$\frac{m}{n} =$

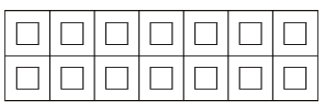
Ib2

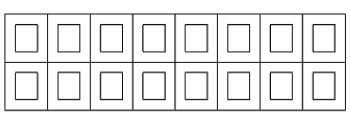
For each chocolate bar shown, find the mass of each smaller piece. Show your solutions.

1.  $m = 120$ g
$\frac{m}{n} =$

2.  $m = 120$ g

3.  $m = 30$ g

4.  $m = 70$ g

5.  $m = 80$ g

Masses of chocolate bar pieces: Procedural

calculation of mass allows numeric comparison

lb1 4-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

Let chocolate bars be composed of identical smaller pieces. The smaller pieces of a given chocolate bar have the same size, shape, and mass. Pieces from different chocolate bars may differ in size, shape, and mass.

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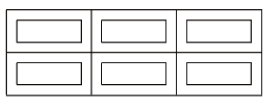
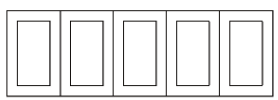
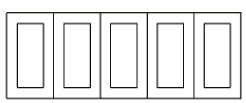


The chocolate bar shown below has a mass of 100 g. It is composed of 5 smaller pieces. Each piece has a mass of 100 g ÷ 5 = 20 g.

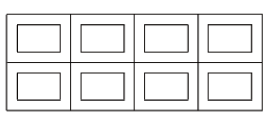


In general, if a chocolate bar has a mass of m and is composed of n smaller pieces, then each piece has a mass of m/n.

For each chocolate bar shown, find the mass of each smaller piece. Show your solutions.

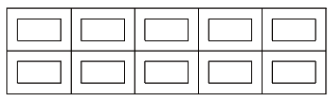
1.  $m = 20$ g

$\frac{m}{n} = \frac{20 \text{ g}}{5} = \square$ g

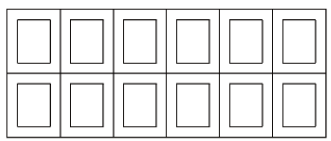
2.  $m = 40$ g

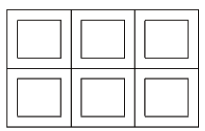
$\frac{m}{n} =$

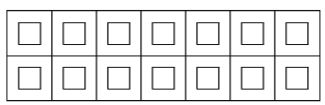
For each chocolate bar shown, find the mass of each smaller piece. Show your solutions.

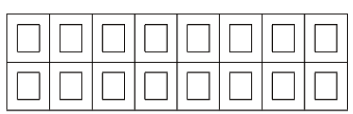
1.  $m = 120$ g

$\frac{m}{n} =$

2.  $m = 120$ g

3.  $m = 30$ g

4.  $m = 70$ g

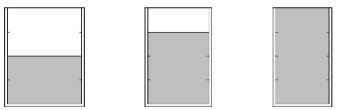
5.  $m = 80$ g

Volumes of liquids in different containers: Conceptual

Ja1 2-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

The three cylindrical containers shown below have the same height but different cross-sectional areas. Each container shown below has the same amount of liquid as the other two containers. Note that the containers have different water levels for the same amount of liquid.



Ja2

The two cylindrical containers shown below contain the same amount of liquid.

1. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

2. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

3. What can be said about the two containers shown below?

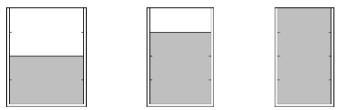
(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Volumes of liquids in different containers: Conceptual

Ja1 2-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

The three cylindrical containers shown below have the same height but different cross-sectional areas. Each container shown below has the same amount of liquid as the other two containers. Note that the containers have different water levels for the same amount of liquid.



conservation of volume

most difficult task is given last

Ja2

The two cylindrical containers shown below contain the same amount of liquid.

1. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

2. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

3. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Volumes of liquids in different containers: Procedural

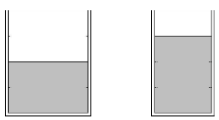
Jb1 2-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

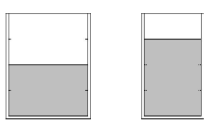
The two cylindrical containers shown below contain liquid and have cross-sectional areas A_1 and A_2, and corresponding liquid levels $h_1 = 2$ and $h_2 = 4$. It is given that the containers have the same volume of liquid. Thus, $h_1A_1 = h_2A_2$, that is, $2A_1 = 4A_2$ or $A_1 = 2A_2$.

Shown below are the same containers but now with different volumes of liquid. The volume of the liquid on the left is $2A_1$; the volume of the liquid on the right is $3A_2$.



Let $2A_1 \,?\, 3A_2$ represent that the relationship between the two volumes is currently unknown. To determine which container has more liquid, start with the given $A_1 = 2A_2$. Now, in $2A_1 \,?\, 3A_2$, replace A_1 with $2A_2$ to get $2(2A_2) \,?\, 3A_2$, that is, $4A_2 \,?\, 3A_2$. But $4A_2 > 3A_2$. Thus, $2A_1 > 3A_2$, that is, the container on the left has more liquid than the one on the right.

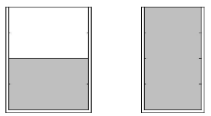
1. Which container has more liquid?



$$A_1 = 2A_2$$
$$2A_1 \,?\, 3A_2$$
$$2(2A_2) \,?\, 3A_2 \qquad 4A_2 > 3A_2$$
$$4A_2 \,?\, 3A_2 \qquad \therefore 2A_1 > 3A_2$$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Jb2



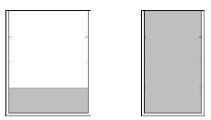
The two cylindrical containers shown contain the same amount of liquid with $2A_1 = 4A_2$, that is, $A_1 = 2A_2$.

1. Which container has more liquid?

$$A_1 = 2A_2$$
$$2A_1 \,?\, 4A_2$$
$$2(2A_2) \,?\, 4A_2 \qquad 4A_2 = 4A_2$$
$$4A_2 \,?\, 4A_2 \qquad \therefore 2A_1 = 4A_2$$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

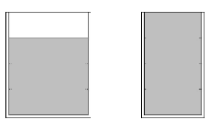
2. Which container has more liquid?



$$A_1 = 2A_2$$
$$A_1 \,?\, 4A_2$$
$$(2A_2) \,?\, 4A_2 \qquad 2A_2 < 4A_2$$
$$2A_2 \,?\, 4A_2 \qquad \therefore A_1 < 4A_2$$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

3. Which container has more liquid? Complete the solution.



$$A_1 = 2A_2$$
$$3A_1 \,?\, 4A_2$$
$$3(2A_2) \,?\, 4A_2 \qquad 6A_2 \bigcirc 4A_2$$
$$6A_2 \,?\, 4A_2 \qquad \therefore 3A_1 \bigcirc 4A_2$$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Volumes of liquids in different containers: Procedural

Jb1 2-20110113

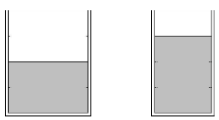
Student number

Date

proportion as an equality of two products of measure (volume is the product of a liquid's height and area)

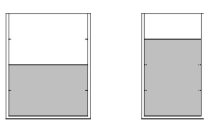
The two cylindrical containers shown below contain liquid and have cross-sectional areas A_1 and A_2, and corresponding liquid levels $h_1 = 2$ and $h_2 = 4$. It is given that the containers have the same volume of liquid. Thus, $h_1 A_1 = h_2 A_2$, that is, $2A_1 = 4A_2$ or $A_1 = 2A_2$.

Shown below are the same containers but now with different volumes of liquid. The volume of the liquid on the left is $2A_1$; the volume of the liquid on the right is $3A_2$.



Let $2A_1 \,?\, 3A_2$ represent that the relationship between the two volumes is currently unknown. To determine which container has more liquid, start with the given $A_1 = 2A_2$. Now, in $2A_1 \,?\, 3A_2$, replace A_1 with $2A_2$ to get $2(2A_2) \,?\, 3A_2$, that is, $4A_2 \,?\, 3A_2$. But $4A_2 > 3A_2$. Thus, $2A_1 > 3A_2$, that is, the container on the left has more liquid than the one on the right.

1. Which container has more liquid?



$A_1 = 2A_2$
$2A_1 \,?\, 3A_2$
$2(2A_2) \,?\, 3A_2$ $\quad 4A_2 > 3A_2$
$4A_2 \,?\, 3A_2$ $\quad \therefore 2A_1 > 3A_2$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

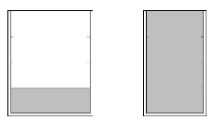
ical containers
e same amount
of liquid with $2A_1 = 4A_2$, that is, $A_1 = 2A_2$.

1. Which container has more liquid?

$A_1 = 2A_2$
$2A_1 \,?\, 4A_2$
$2(2A_2) \,?\, 4A_2$ $\quad 4A_2 = 4A_2$
$4A_2 \,?\, 4A_2$ $\quad \therefore 2A_1 = 4A_2$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

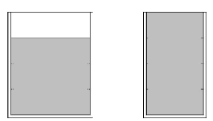
2. Which container has more liquid?



$A_1 = 2A_2$
$A_1 \,?\, 4A_2$
$(2A_2) \,?\, 4A_2$ $\quad 2A_2 < 4A_2$
$2A_2 \,?\, 4A_2$ $\quad \therefore A_1 < 4A_2$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

3. Which container has more liquid? Complete the solution.



$A_1 = 2A_2$
$3A_1 \,?\, 4A_2$
$3(2A_2) \,?\, 4A_2$ $\quad 6A_2 \bigcirc 4A_2$
$6A_2 \,?\, 4A_2$ $\quad \therefore 3A_1 \bigcirc 4A_2$

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Review of worksheets D, E, F, G, H, I, and J: Conceptual

Ka1 2-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

For each item below, write $<$, $>$, or $=$ in the circled space.

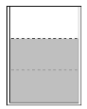
1. $\frac{2}{5}$ ◯ $\frac{3}{5}$
2. $\frac{3}{5}$ ◯ $\frac{3}{4}$
3. $\frac{2}{5}$ ◯ $\frac{3}{4}$

For each item below, fill in the blank with a number that makes the given relationship true.

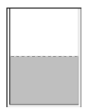
1. $\frac{3}{5} < \frac{\square}{5}$
2. $\frac{3}{5} > \frac{\square}{5}$
3. $\frac{3}{5} < \frac{3}{\square}$
4. $\frac{3}{5} > \frac{3}{\square}$
5. $\frac{5}{9} < \frac{5}{\square} < \frac{5}{7}$
6. $\frac{5}{9} > \frac{\square}{9} > \frac{3}{9}$
7. $\frac{4}{7} < \frac{\square}{\square} < \frac{5}{6}$
8. $\frac{4}{7} > \frac{\square}{\square} > \frac{3}{8}$

Ka2

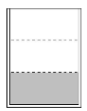
1. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



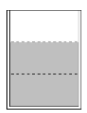
2. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?



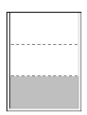
3. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?



4. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



5. 2 L of a certain liquid weigh 3 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



Review of worksheets D, E, F, G, H, I, and J: Conceptual

Ka1

2-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____

Date: _____ Time finished: _____

For each item below, write $<$, $>$, or $=$ in the circled space.

1. $2/5$ ◯ $3/5$
2. $3/5$ ◯ $3/4$
3. $2/5$ ◯ $3/4$

highly sequential order builds upon previously learned concepts

$(\frac{2}{5} < \frac{3}{5}, \frac{3}{5} < \frac{3}{4}, \frac{2}{5} < \frac{3}{4})$

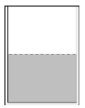
For each item below, fill in the blank with a number that makes the given relationship true.

1. $3/5 < \square/5$
2. $3/5 > \square/5$
3. $3/5 < 3/\square$
4. $3/5 > 3/\square$
5. $5/9 < 5/\square < 5/7$
6. $5/9 > \square/9 > 3/9$
7. $4/7 < \square/\square < 5/6$
8. $4/7 > \square/\square > 3/8$

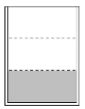
Ka2

1. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?

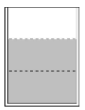
ain liquid weighs 1 1/2 kg. When full, r shown at the right holds 2 L. How ms does the liquid shown at the right



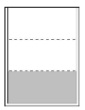
3. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L. How many kilograms does the liquid shown at the right weigh?



4. 1 L of a certain liquid weighs 1 1/2 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



5. 2 L of a certain liquid weigh 3 kg. When full, the container shown at the right holds 3 L. How many kilograms does the liquid shown at the right weigh?



Review of worksheets D, E, F, G, H, I, and J: Procedural

Kb1 3-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

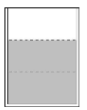
1. $^1/_4$ ◯ $^2/_7$ $\quad 1 \times 7 \textcircled{?} 2 \times 4$, $7 < 8$
2. $^2/_4$ ◯ $^3/_7$
3. $^3/_4$ ◯ $^5/_7$
4. $^5/_8$ ◯ $^2/_3$
5. $^6/_8$ ◯ $^3/_4$
6. $^7/_8$ ◯ $^5/_6$

Kb2

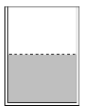
Find the weight in kilograms of the liquid shown at the right. Show your solutions.

1. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.

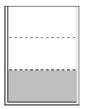
$$m_2 = \frac{m_1}{v_1} v_2 = \frac{2 \text{ kg}}{1 \text{ L}} \cdot \frac{2}{3}(3 \text{ L}) = \square \text{ kg}$$



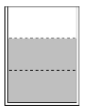
2. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 2 L.



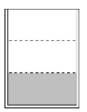
3. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.



4. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 3 L.



5. 2 L of a certain liquid weigh 3 kg. When full, the container shown at the right holds 3 L.



Review of worksheets D, E, F, G, H, I, and J: Procedural

Kb1 3-20110113

Student number: ☐☐☐☐-☐-☐☐☐☐ Time started: _____
Date: _____ Time finished: _____

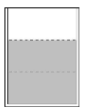
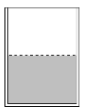
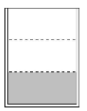
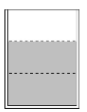
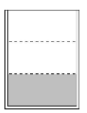
For each numbered item below, compare the values of the two fractions $^a/_c$ and $^b/_d$ by comparing the values of the products ad and bc. Show your solutions. Write $<$, $>$, or $=$ in the circled space.

1. $^1/_4$ ◯ $^2/_7$ $\quad 1 \times 7 \,\textcircled{?}\, 2 \times 4$, $7 < 8$
2. $^2/_4$ ◯ $^3/_7$
3. $^3/_4$ ◯ $^5/_7$
4. $^5/_8$ ◯ $^2/_3$
5. $^6/_8$ ◯ $^3/_4$
6. $^7/_8$ ◯ $^5/_6$

hints for earlier topics

Kb2

Find the weight in kilograms of the liquid shown at the right. Show your solutions.

1. 1 L of a certain liquid weighs 2 kg. When full, the container shown at the right holds 3 L.
$$m_2 = \frac{m_1}{v_1} v_2 = \frac{2 \text{ kg}}{1 \text{ L}} \cdot \frac{2}{3}(3 \text{ L}) = \square \text{ kg}$$

2. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 2 L.

3. 1 L of a certain liquid weighs 1 kg. When full, the container shown at the right holds 2 L.

4. 1 L of a certain liquid weighs 1 ½ kg. When full, the container shown at the right holds 3 L.

5. 2 L of a certain liquid weigh 3 kg. When full, the container shown at the right holds 3 L.


Results and Recommendations

Undergraduates assigned the procedural version of the worksheets had significantly longer completion times than those assigned the conceptual version of the worksheets.

These worksheets were used to find empirical evidence on the causal relationships between conceptual and procedural knowledge, so the two sets of worsheets should differ only in the type of instruction and not in the duration of instruction.

The worksheets need to be further revised and tested to finally determine whether or not there is a significant difference in completion times between the two versions.

Additional worksheets are also being planned.

End